Vertical Reference Frames in Slovakia and their Reciprocal Differences

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ABSTRACT

Numerical expression of vertical differences among vertical reference frames used in Slovakia and neighbouring countries requires common practice based on the need of exchange of geodetical data within different international projects, where different international institutions take part in. The presented article describes definitions and realizations of three vertical reference frames: Adriatic (zero levelling point in Trieste, Italy), Baltic after adjustment – Bpv (zero levelling point in Kronstadt, Russia) and European (zero levelling point in Amsterdam, The Netherlands), transformation relations among them and numerical computation of vertical differences among them.

Keywords: vertical reference systems, vertical reference frames, transformation, geoid, quasi-geoid

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1. PREFACE

New theoretical knowledges about earth´s structure and its physical characteristics have large impact on definitions and realizations of vertical reference frames (*Drewes*, *2009*). Real implementation of a vertical reference system on the millimetre level of precision constitutes highly ambitious task regarding its theoretical formulation, numerical realization and computation of the required corrections during processing of measured data.

Generally, vertical reference system is defined by dataset of constants, a model of mean sea level heights and by tidal model.

Vertical reference frame is realization of a defined vertical reference system.

In Europe, there are three different kinds of physical heights (mean sea level heights – MSL): *normal* MSL heights, *orthometric* MSL heights, *normal orthometric* MSL heights. Normal heights are used in France, Germany, Sweden and in most central and eastern European countries, orthometric heights are used in Belgium, Denmark, Finland, Italy and Switzerland, normal orthometric heights are used in Norway, Austria and in the Balkans countries.

Further in text we will alternate the use of mean sea level phrase and its abbreviation MSL. When it will not be otherwise stated explicitly, the word *height* will always be related to the height above mean sea level (MSL).

Vertical reference frame on earth´s surface is realized by mean sea level, which is monitored on one or several stations with water level gauges. Reference water level gauges are installed in oceans or inland seas like Atlantic Ocean, Baltic Sea, Adriatic Sea, Black Sea or Northern Sea. Differences among their mean sea levels can vary between several decimetres up to meter level (e.g. around Australia). This is caused by different mean sea level height of oceans and seas with respect to physical centre of earth (*Ihde and Augath, 2000*).

Current solution of tasks bounded to vertical reference frames requires information about dynamics of earth´s surface in static, kinematic and dynamic shape. It supposes deeper study of theoretical and experimental results of the whole process, which run all over earth´s surface, where is planned the establishment of a vertical reference network for monitoring of vertical variations.

2. BASIC DEFINITIONS AND INFORMATION

A vertical reference frame is a single-dimensional coordinate system defined by an origin and by scale defined by levelling rod. The origin is a point on the earth´s surface, where its height is defined by a specific value or by mean height of tide gauge. The height differences can be measured from this origin to any point using standard levelling procedures and corrections complying the parameters of actual or normal earth´s gravity field. Mean sea level point traditionally served as an origin. The absolute value of the earth´s gravity field is not important on this point, because we are interested in only vertical differences (potential differences) with respect to the origin (*Jekeli, 2006*), see Fig.1.

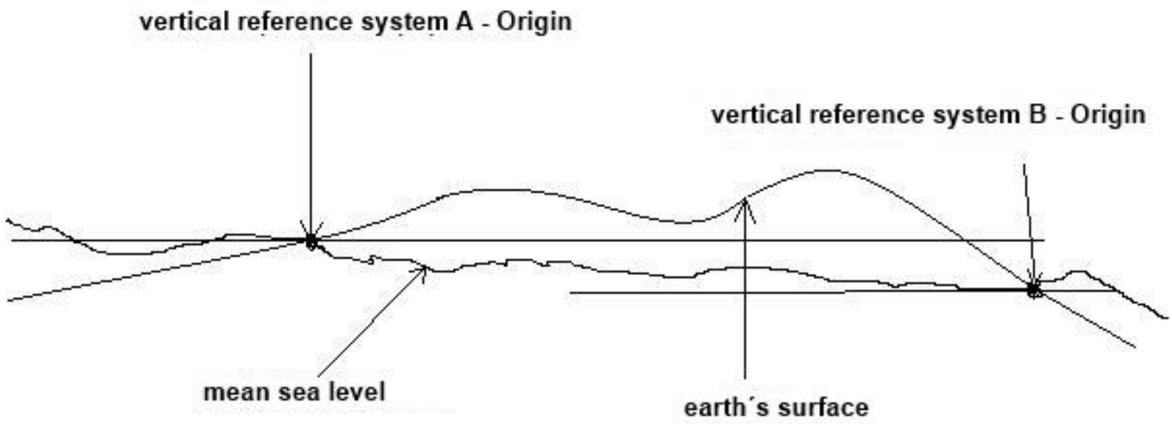


Fig.1. Two vertical reference frames.

2.1 Adriatic vertical reference frame

Adriatic vertical reference frame is defined by mean sea level of Adriatic Sea measured in Trieste, Italy (Molo Sartorio) in 1875 by Dr. Farolfi using measurements of sea level during one-year period only. The measured height differences were corrected using normal orthometric corrections. The adjustment of the levelling network of the 1st order was carried out by means of conditional equations (for more details – see *Vykutil, 1978*).

However, the epoch, in which Dr. Farolfi measured the mean sea level of Adriatic Sea was too short. Considering the positional variations of the Moon and the Sun with respect to the Earth, it was needed to observe the position of the Adriatic Sea level during period of 18.6 year (*Lowrie, 2007*). Primary vertical point was stabilised on the custom house in Trieste, Italy. The levelling lines were measured from this point over the whole former Austro-Hungarian monarchy and nearly exclusively along railroads.

After 1945, there was established the Czecho-Slovak unified levelling network (**<u>č</u>**esko-**<u>s</u>**lovenská **<u>j</u>**ednotná **<u>n</u>**ivelačná **<u>s</u>**ieť), in short referred to as ČSJNS (this abbreviation will be used in the following text regularly) with its basic height point Lišov (near the town České Budejovice in the Czech republic). The results of measurements of levelling lines carried out during 1939 – 1941 in the Czech Republic were incorporated into ČSJNS of the 1st order along with new levelling lines measured during 1946 – 1948 in Czecho-Slovak border regions and during 1949 – 1952 in Slovakia. The ČSJNS of the 1st order was adjusted as a whole after applying normal orthometric corrections.

2.2 Baltic vertical reference frame after adjustment – Bpv

Baltic vertical reference frame after adjustment (Baltský výškový referenčný rámec - **<u>B</u>**alt **<u>p</u>**o **<u>v</u>**yrovnaní), in short referred to as Bpv (this abbreviation will be used in the following text regularly) is defined by mean sea level of Baltic Sea in Kronstadt on the Kotlin island, near Sankt Petersburg, Russia. The measured height differences were corrected by normal earth´s gravity field corrections according to the theory of Molodensky prior to adjustment. In 1957, the ČSJNS levelling network of the 1st order was jointly adjusted along with the levelling networks of western part of then Soviet Union and also with levelling networks of former East Germany, Poland, Hungary and Bulgaria.

On the territory of former Czecho-Slovakia, the mean sea level heights in the vertical reference frame Bpv differ from Adriatic mean sea level heights in the interval of **<u>35 – 42 cm</u>**. The reason for the differences is due to the:

- set up of different mean sea level heights for both vertical reference systems
- use of different method of computation of earth´s gravity field corrections
- international adjustment of levelling network.

For example, the mean sea level height of the point Lišov (near the town České Budejovice in the Czech republic) in the Bpv vertical reference frame is 564.7597 m, what is in 0.3886 m lower than in Adriatic vertical reference frame (see *Vykutil, 1978; Böhm, Hora, Kolenatý 1981)*.

This approach is based on the stationary principle of earth´s surface and oceans and assumes that there is no change on earth´s surface and that there is no change in the mean sea level height in the course of time.

However, up-to-date knowledges regarding height measurements on the earth´s surface and mean sea level measurements confirm that there are changes in heights on the earth´s surface and also there are changes in mean sea level heights in water level gauges in the course of time, first of all due to the:

1. melting of icebergs on the whole earth´s surface (mostly in Greenland and Antarctica). This results in the change of mean sea level heights in water level gauges on average in 3.2 mm a year,
2. losing of substances of icebergs which are moved into the oceans. This results in the so-called global isostatic adjustment of earth´s surface. In Europe, this is expressed by global isostatic lift of earth´s surface in Scandinavian area (about 1 cm a year). In

Slovakia, global isostatic adjustment shows drop of earth´s surface on the level of approximately 1 mm a year.

These effects, however, are not significant for relative changes of height differences between vertical reference frames **Adria**$_{\textbf{ČSJNS}}$ (also known as **Jadran**$_{\textbf{ČSJNS}}$) and **Bpv**.

2.3 European vertical reference system (EVRS)

European vertical reference system (EVRS) was established based on the initiative of the EUREF commission (European regional commission of the International geodetical association for geodetical reference systems) with the aim to build up a united European levelling network (UELN-United European Levelling Network). UELN was established in 1973 and contained levelling networks of the 1st order of 14 western European countries.

In 1994, during EUREF session in Warsaw, Poland, a resolution was adopted to join vertical reference frames of all European countries with an accuracy better than 0.1 m (*Beneš, 1999*). The realization of the process was carried out in a few phases. Last phase was branded as UELN-95/98 and constitutes a base for a European vertical reference system adopting all future measurements.

European vertical reference system (EVRS) is defined by the following conventions:

a) Vertical reference system has a zero value in a point, in which actual earth´s gravity potential W_o equals normal gravity potential U_o on a mean earth´s ellipsoid:

$$W_o = U_o \,, \qquad (1)$$

b) Height component is basically the difference ΔW_P between earth´s field gravity potential in the point P and earth´s field gravity potential W_o (gravity potential on the geoid surface). The difference of potential - ΔW_P is referred to as geopotential number c_P

$$-\Delta W_P = W_o - W_P = c_P \,, \qquad (2)$$

Normal heights are equivalent to geopotential numbers.

c) EVRS (European Vertical Reference System) parameters are expressed in zero tidal system. It means that potentials both in the definition a) and in the definition b) contain potentials of permanent tidal deformations, however – they do not contain permanent tidal potential itself.

2.4 European vertical reference frame 2000 (EVRF2000)

Realization of the European vertical reference frame 2000 (*EVRF2000, see Ihde, Mäkinen., Sacher, 2007*) comes out of the following adopted principles:

1. Zero level gets across a reference point of the UELN network No. 000A2530 – Normaal Amsterdams Peil (NAP) located in Amsterdam, The Netherlands. It means that it is valid

$$C_{NAP} = 0. \tag{3}$$

2. For the computation of derived parameters and constants of the normal earth´s gravity field is used Geodetical Reference System 1980 (GRS80). It follows that

$$W_{NAP}^{RREAL} = U_{oGRS80}. \tag{4}$$

3. European vertical reference frame 2000 (EVRF2000) is fixed by geopotential number of 7.0259 $m^2.s^{-2}$ and by equivalent normal height of 0.71599 m on the reference point NAP No. 000A2530 in Amsterdam, The Netherlands.
4. EVRF2000 is realized for practical use as a static vertical reference frame, defined by means of differences of earth´s gravity field potentials with respect to the NAP reference point or by equivalent normal heights of points with respect to the NAP reference point.
5. Geometrical position of points is expressed in the European Terrestrial Reference Frame 1989 (ETRF89).
6. EVRF2000 uses the Zero Tide System for the reduction of measured data.

The EVRF2000 can be realized by two techniques or methods:

a) By means of levelling and gravimetric observations and subsequently by adjustment of geopotential numbers as follows:

$$W_P = W_o - c_{p,} Hn = c_p / \bar{\Upsilon}, \tag{5}$$

where $\bar{\Upsilon}$ is an average value of normal acceleration of gravity along of normal plumb line from equipotential ellipsoid to the telluroid,

b) By means of GNSS observations (ellipsoidal height h_P) and by global quasi-geoid model (Global Gravity Model - GGM) by solving the geodetic boundary value problem (GBVP)

$$W_P = U_P + T_P, \zeta_P = T_P/\Upsilon_Q = (W_P - U_P)/\Upsilon_Q, Hn = h_P - \zeta_P, \tag{6}$$

where ζ_P is the height (undulation) of a geoid or a quasi-geoid in the point P.

2.5 European vertical reference frame 2007 (EVRF2007)

EVRF2007 is a new realization of the EVRS based on the combined strategy of three elements: reference network, vertical reference frame and observations developing in the course of time of the reference frame. If it is possible, the measured data are reduced to the epoch 2000 to enable the determination of the transformation relationships between EVRF2007 and EVRF2000. EVRF2007 is realized by a new adjustment of the UELN by means of geopotential numbers. All measurements from Scandinavian states are reduced to the epoch 2000 from post glacial rebound. The reference frame EVRF2007 is realized using 13 reference points on European continent with special stabilization and their geopotential differences comply to the following condition:

$$\sum_{i=1}^{13}(c_{P,95/98} - c_{P,EVRF2007}) = 0. \quad (7)$$

Geopotential differences are reduced to zero tidal system. All details regarding the organization of observations, reductions and the adjustment of the observations can be found in (Sacher, Liebsch, 2015).

2.6 Mathematical formulation of the difference between two vertical reference frames

Let´s label the reference height in the Adriatic vertical reference frame (**Jadran**$_{\textbf{ČSJNS}}$) by superscript A and in the Baltic vertical reference frame after adjustment (**Bpv**) by superscript B (see Fig. 1). Let´s consider two points C and D, which have known reference heights in both vertical reference frames A and B. The reference height of the point D in the **Jadran**$_{\textbf{ČSJNS}}$ can be expressed by the following equation:

$$H^A(D) = H^A(C) + \Delta h_{CD}^{niv} + C_{CD}^1 + v_D, \quad (8)$$

where

$H^A(D)$ - is reference height of the point D in **Jadran**$_{\textbf{ČSJNS}}$ in meters,

$H^A(C)$ - is reference height of the point C in **Jadran**$_{\textbf{ČSJNS}}$ in meters,

Δh_{CD}^{niv} - is levelling height difference between points C and D in meters,

C_{CD}^1 - is normal orthometric correction between points C and D in millimetres,

v_D^A - is correction coming out of the adjustment in **Jadran**$_{\textbf{ČSJNS}}$ on the point D in millimetres.

The normal orthometric correction C_{CD}^1 between points C and D can be computed from the equation (*Pick, 1987*)

$$C_{CD}^1 = -0.0000254\, H_m \Delta\varphi'' \text{ (in mm)}, \quad (9)$$

where

$H_m = \dfrac{H_C + H_D}{2}$ (in meters),

$\Delta\varphi$ is in angular seconds.

For the reference height of the point D determined in the vertical reference frame **Bpv** can be written the following equation:

$$H^B(D) = H^B(C) + \Delta h_{CD}^{niv} + C_{CD}^1 + C_{CD}^2 + v_D^B,$$
(10)

where

$H^B(D)$ - is reference height of the point D in **Bpv** in meters,

$H^B(C)-$ is reference height of the point C in **Bpv** in meters,

Δh_{CD}^{niv} - is levelling height difference between points C and D in meters,

C_{CD}^2- is earth´s field gravity correction between points C and D in millimetres,

v_D^B- is correction coming out of the adjustment in **Bpv** on the point D in millimetres.

The earth´s field gravity correction C_{CD}^2 between two points C and D can be computed from the equation (*Pick, 1987*)

$$C_{CD}^2 = 0.0010193\,(g-\gamma)_m \Delta h_{CD}^{mer} \quad \text{(in mm).}$$
(11)

where

$(g-\gamma)_m$ - is mean value of the Fay´s anomaly of the acceleration of gravity between points C and D in milligals,

Δh_{CD}^{mer} - is levelling height difference between points C and D in meters.

The Fay´s anomalies of the acceleration of gravity are unsuitable for the use of linear interpolation method in the practical calculations. If there is available a map of Bouger´s anomalies of acceleration of gravity $(g-\gamma)_B$, which is sufficiently precise, the interpolated values of Bouguer's anomalies of acceleration of gravity can be transformed on the Fay´s anomalies of acceleration of gravity by means of equation

$(g-\gamma)_F = (g-\gamma)_B + 0.1119H$ (in milligals),
(12)

where H is mean sea level height of the point in meters.

If we subtract the equation (8) from the equation (10), we get the ***height difference between two vertical reference frames*** and in our case it means the height difference between Baltic vertical reference frame after adjustment (Bpv) and Adriatic vertical reference frame ($\text{Jadran}_{\text{ČSJNS}}$) in the point D:

$$\Delta H^{AB}(D) = H^B(D) - H^A(D) = H^B(C) + \Delta h_{CD}^{niv} + C_{CD}^1 + C_{CD}^2 + v_D^B - H^A(C) - \Delta h_{CD}^{niv} - C_{CD}^1 - v_D^A, \tag{13}$$

eventually

$$\Delta H^{AB}(D) = H^B(D) - H^A(D) = H^B(C) - H^A(C) + C_{CD}^2 + v_D^B - v_D^A$$
(14)

The equation (14) can be used for control calculation of the height differences between two vertical reference frames on any point, if we know the height difference between two vertical reference frames on a specific point and we apply the earth´s field gravity correction C^2 between specific point and an arbitrarily chosen point. The calculation of $v_D^B - v_D^A$ values is problematic in such case, when the corrections coming out of a vertical reference network adjustment are unknown, what is also our case.

We assume that the corrections coming out of the vertical reference network adjustment are sufficiently small and do not exceed the value of 10 mm. This is the reason, why we can neglect them (*Vykutil, 1978*).

3. Computation of height differences on levelling points with historical Adriatic heights (JadranČSJNS) and historical Baltic heights after adjustment (Bpv) along the Austrian – Slovakian border on the Morava river

3.1 Historical background of transition from JadranČSJNS to Bpv

Adriatic vertical reference frame realized by ČSJNS (**JadranČSJNS**) was a mandatory vertical reference frame within former Czecho-Slovakia till 1952. For topographic mapping of the scale 1:25 000, which started in 1952, the mean sea level heights of points in the Adriatic vertical reference frame **JadranČSJNS** were transformed into approximate Baltic vertical reference frame by subtracting a constant value of 0.68 m. This Baltic vertical reference frame was referred to as **B-68**.

As of February 17, 1955, by decision of the former Main Executive of Geodesy and Cartography (Ústredná správa geodézie a kartografie – ÚSGK) was introduced an interim Baltic vertical reference frame referred to as **B-46**. All mean sea level heights of levelling points in this interim vertical reference frame B-46 were computed by subtracting a constant value of 0.46 m from mean sea level heights determined in Adriatic vertical reference frame **Jadran**$_{\check{C}SJNS}$. The height difference (46 cm) was computed on an identical levelling point of former Czecho-Slovakian and soviet levelling network in the border village Čierna nad Tisou located on the current Slovakian and Ukrainian border.

In 1957, the former Czecho-Slovakian levelling network of the 1st order was collectively adjusted along with levelling network of western part of then Soviet Union and also with the levelling networks of former socialist countries. Prior to adjustment, all observed height differences were corrected by Molodensky´s corrections coming out of actual acceleration of gravity observed on the earth´s surface. The basic point (a point with zero mean sea level height) for this adjustment was a point with tide gauge in Kronstadt on the Kotlin island, near Sankt Peterburg, Russia. So emerged the Baltic vertical reference frame after adjustment also known as Bpv (Baltský výškový systém po vyrovnaní – Bpv, see Vykutil, 1978).

The computation of the height differences between historic Adriatic vertical reference frame realized by levelling network of the 1st order **Jadran**$_{\check{C}SJNS}$ (1951) and historic Baltic vertical reference frame after adjustment **Bpv** (1951) was carried out in the middle of the Morava river which constitutes the border between Austria and Slovakia, starting at the confluence of rivers Dyje and Morava on the Austria-Czech-Slovakian border point as far as the confluence of rivers Morava and Danube at the Devín village in Slovakia, near Slovakian capital Bratislava and thereof in the middle of Danube river as far as Lafranconi bridge in Bratislava, for the purposes of Slovak Water Management Enterprise company, state enterprise with headquarters in Banská Štiavnica, subsidiary Závod Bratislava. The computation was carried out based on the data which were provided by Geodetical and Cartographical Institute, Bratislava (Geodetický a kartografický ústav Bratislava - GKÚ) – see Tab. 1 and Tab. 2.

Subsequently, using the same method of computation in the same geographical area, the determination of height differences was carried out between currently mandatory Adriatic vertical reference frame (Adria) in Austria and currently mandatory Baltic vertical reference frame after adjustment (Bpv) in Slovakia, by using currently valid Adriatic mean sea level heights on 57 Austrian levelling points located on the Austrian side of the Morava river and also by using 52 Slovakian levelling points with currently valid Baltic after adjustment mean sea level heights on Slovakian side of the Morava river. Both sets of levelling points have their mean sea level heights determined by precise levelling method in their respective vertical reference frames – Adria and Bpv (see Chapter 4).

3.1.1 Important explaining comment regarding the use of Slovakian levelling points with their historical mean sea level heights in both vertical reference frames Jadran$_{\check{C}SJNS}$ and Bpv

To perform internally consistent height transformation from one vertical reference frame to the another one and vice versa (in our case in this Chapter 3, we are dealing with height transformation between **Jadran**$_{\check{C}SJNS}$ **(1951)** a **Bpv (1951)** on Slovakian levelling points only), it is necessary to know the mean sea level heights on a sufficient set of identical vertical points in the area of interest in both considered vertical reference frames. The observations should come out of the same or nearly the same time period. To solve the task, we have to take into our consideration the fact that we have available only historical mean sea level heights in the Adriatic vertical reference frame **Jadran**ČSJNS on Slovakian levelling points (originating from 1951 and since then these **Jadran**ČSJNS mean sea level heights on Slovakian levelling points were never updated, since this vertical reference frame is no more used in Slovakia). It practically means that we also have to take into the computation of the mean sea level heights in the Baltic vertical reference frame after adjustment (**Bpv**) on the same set of Slovakian levelling points originating from the same time period.

If we would use currently valid mean sea level heights in vertical reference frame **Bpv**, the solution would inherently be inconsistent, because we would compare the currently valid mean sea level heights in the vertical reference frame **Bpv** with historical (more than 60 years old) mean sea level heights in the Adriatic vertical reference frame **Jadran**ČSJNS **(1951,** which had to be considered as static (unvarying), what is in deep contrast with the reality of geodynamic phenomenon (ascending and declining) of the earth´s surface in the course of time. This fact cannot be neglected for longer time period. It can clearly be seen, when we look on the mean sea level heights on the identical Slovakian levelling points of the 1st order in the vertical reference frame Bpv, which are valid in 2018 (see Tab. 1, 4th column from the left). If we would compute the height differences between the **Jadran**ČSJNS **(1951)** vertical reference frame and the **Bpv (2018)** vertical reference frame, on some points these height differences would be very close to the height differences between **Jadran**ČSJNS **(1951)** a **Bpv (1951)** – for example, point CZA-544, where the difference of height differences is 0.0001m. On other points are the differences greater (several cm). The significant difference is in our case on the identical levelling point ZBZC-544, where the difference of the height differences reaches the value of 0.1078 m (it is in the area Sekule – Brodské, state border between Slovakia and the Czech Republic).

This is the reason, why we use the abbreviation **Bpv (1951)** in our text. It is necessary to understand it in such manner that we are speaking about mean sea level heights or their differences on identical levelling points which were observed in 1949 – 1951 (*Kruis, 1957*) and adjusted in the first adjustment within vertical reference frame Bpv in 1957. From this year onward, the vertical reference frame Bpv was officially adopted as the only mandatory vertical reference frame in Slovakia. The considered area of interest (about 78.3 km long line running in the middle of the Morava and Danube river) was divided on 314 small segments (each of them 250 m long) with planar horizontal coordinates X,Y in Slovakian national horizontal coordinate frame referred to as S-JTSK (Systém jednotnej trigonometrickej siete katastrálnej or System of the unified trigonometrical cadastral network) as can be seen on the Fig. 2, which represents small part (confluence of Morava and Danube river) of the total 78.5 km length.

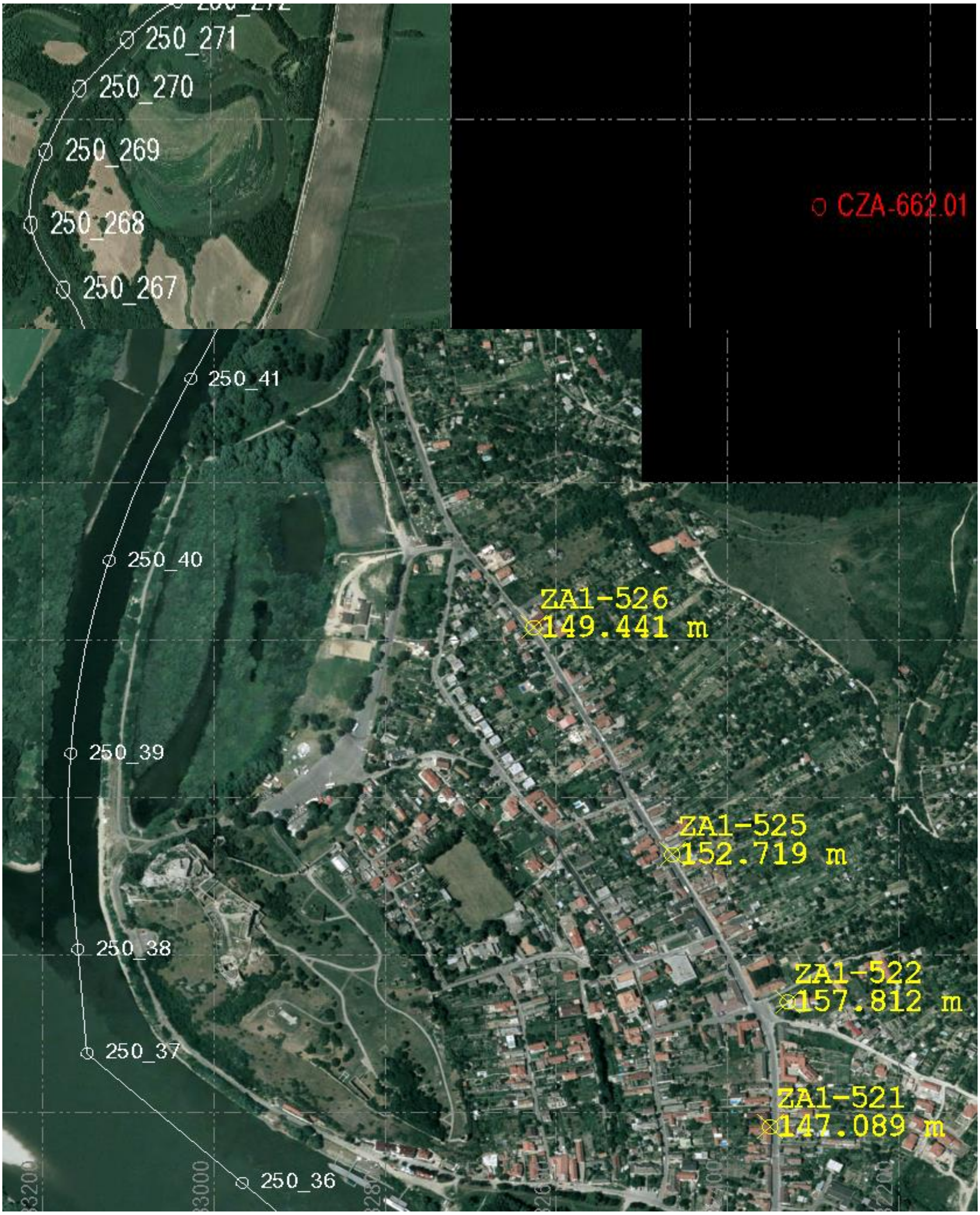

250_271
250_270
250_269
250_268
250_267
CZA-662.01
250_41
250_40
250_39
250_38
250_37
250_36
ZA1-526
149.441 m
ZA1-525
152.719 m
ZA1-522
157.812 m
ZA1-521
147.089 m

Fig.2. Segmentation in the middle of Morava/Danube river with points (each other away 250m – in white). Nearby points in yellow colour represent levelling points of the 2nd order levelling network and in red colour are points of the 1st order levelling network.

The points in white colour in the Fig. 2 represents start/end point of 250 m long segment with their horizontal coordinates in the Slovakian national coordinate frame S-JTSK. On these points are computed the mean sea level differences between vertical reference frames **JadranČSJNS (1951)** and **Bpv (1951)**.

The red coloured points in the Fig. 2 are points of the Slovakian levelling network of the 1st order. They have known their mean sea level heights in both vertical reference frames **Bpv (2018)** a **JadranČSJNS (1951)** along with their known mean sea level differences between vertical reference frames **JadranČSJNS (1951)** and **Bpv (1951) –** see Tab. 1 (source of the data – Geodetical and Cartographical Institute Bratislava).

Tab. 1 Geodetical data of points of the 1st order levelling network in **JadranČSJNS** a **Bpv** (all horizontal and vertical values are in meters).

Point ID	**JadranČSJNS (1951)**	**Difference JadranČSJNS (1951) – Bpv (1951)**	**Bpv (valid in 2018)**	**Year of observation**	**Year of inspection**	**Y (S-JTSK) (Easting)**	**X (S-JTSK) (Northing)**
CZA-520	189.0396	0.4022	188.6289	3.7.2000	2000	575144.40	1278448.87
CZA-544	218.2093	0.4011	217.8081	2.6.1993	2002	577690.00	1275092.81
CZA-662.01	151.7957	0.4019	151.3466	23.9.1992	2000	578462.14	1237279.37
CZA-673	152.6300	0.4019	152.1968	3.7.2000	2000	577324.40	1232601.36
ZBZC-544	154.6547	0.4017	154.1452	5.5.1999	1999	576333.79	1219729.46

The yellow coloured points are points of the Slovakian levelling network of the 2nd order. They have known their mean sea level heights in vertical reference frame **Bpv (2018)** and the mean sea level differences between vertical reference frames **JadranČSJNS (1951)** a **Bpv (1951)** are also known on these points – see Tab. 2 (source of data – Geodetical and Cartographical Institute Bratislava).

Tab. 2 Geodetical data of points of the 2nd order levelling network (all horizontal and vertical values are in meters).

Point ID	**Bpv (2018)**	**JadranČSJNS (1951) – Bpv (1951)**	**Y (S-JTSK) (Easting)**	**X (S-JTSK) (Northing)**
ZBZC-539	155.0229	0.4017	576105.64	1220244.48
ZBZC-540	154.6666	0.4017	576293.36	1219901.90
ZBZC-541	154.6824	0.4017	576331.94	1219830.29

ZBZC-544	154.1453	0.4017	576333.79	1219729.46
ZA18-511.02	145.4447	0.4019	588055.69	1243611.38
ZA18-513	145.7287	0.4020	588021.15	1244552.74
ZA18-513.02	145.6217	0.4020	588271.87	1244870.79
ZA18-530	146.4116	0.4023	589184.73	1249733.66
ZA18-532	145.3157	0.4023	589261.04	1249883.44
ZA10-517	145.3348	0.4026	586200.15	1259252.89
ZA10-518	144.6217	0.4026	586324.41	1259080.03
ZA10-520	145.0833	0.4025	586408.11	1258685.29
ZA2-501	150.9765	0.4034	582970.64	1272512.62
ZA2-502	151.9138	0.4034	582866.73	1272181.68
ZA2-503	143.4469	0.4036	582803.78	1271952.21
ZA1-505	140.7220	0.4038	578371.14	1279829.77
ZA1-510	143.1379	0.4037	579805.54	1279212.52
ZA1-516	152.5220	0.4033	581033.45	1278457.07
ZA1-521	147.0887	0.4035	582349.61	1276818.83
ZA1-522	157.8125	0.4032	582329.15	1276659.97
ZA1-525	152.7186	0.4034	582464.34	1276472.47
ZA1-526	149.4408	0.4035	582626.21	1276183.70
ZA1-537	142.2484	0.4036	583167.77	1273182.85
ZA1-539	145.5621	0.4035	583134.29	1272905.53
ZA2-500	149.7181	0.4034	583008.26	1272776.10
CZA-501	141.5229	0.4038	572785.25	1279759.34
CZA-503	152.7604	0.4035	573295.73	1279656.07
CZA-505	159.6769	0.4033	573585.90	1279603.79
CZA-506	162.8133	0.4032	573568.09	1279381.32
CZA-508	166.2824	0.4030	573504.27	1279260.68
CZA-509	176.0520	0.4027	573547.03	1279169.67

In the 3rd column of the Tab. 1 and Tab. 2 are presented the mean sea level differences between vertical reference frames **Jadran$_{\text{ČSJNS}}$ (1951)** and **Bpv (1951)** originating from mean sea level heights in both vertical reference frames of that time period (around the year 1951) as provided by Geodetical and Cartographical Institute Bratislava.

To verify the correctness of the determination of the mean sea level differences between two vertical reference frames on any point in the area of interest, we used 5 points of the 1st order levelling network (ČSJNS), which are located nearby (less than 10 km away from the rivers Morava and Danube). In the Tab. 1 are presented their horizontal and vertical data along with the mean sea level differences between **Jadran$_{ČSJNS}$ (1951)** a **Bpv (1951)**.

3.1.2 Verification of the computation of the vertical mean sea level differences between two vertical reference frames

As can be seen in the map of full Bouguer's anomalies of acceleration of gravity in Slovakia (Fig. 3), (source – Slovak Academy of Science, Slovenská akadémia vied – SAV) and of a part of Danube lowland (Fig. 4) as presented by (*Milička, 2017*), we estimated the Bouguer´s anomalies for all 5 points of the 1st order levelling network and the estimations are illustrated in the Tab. 3.

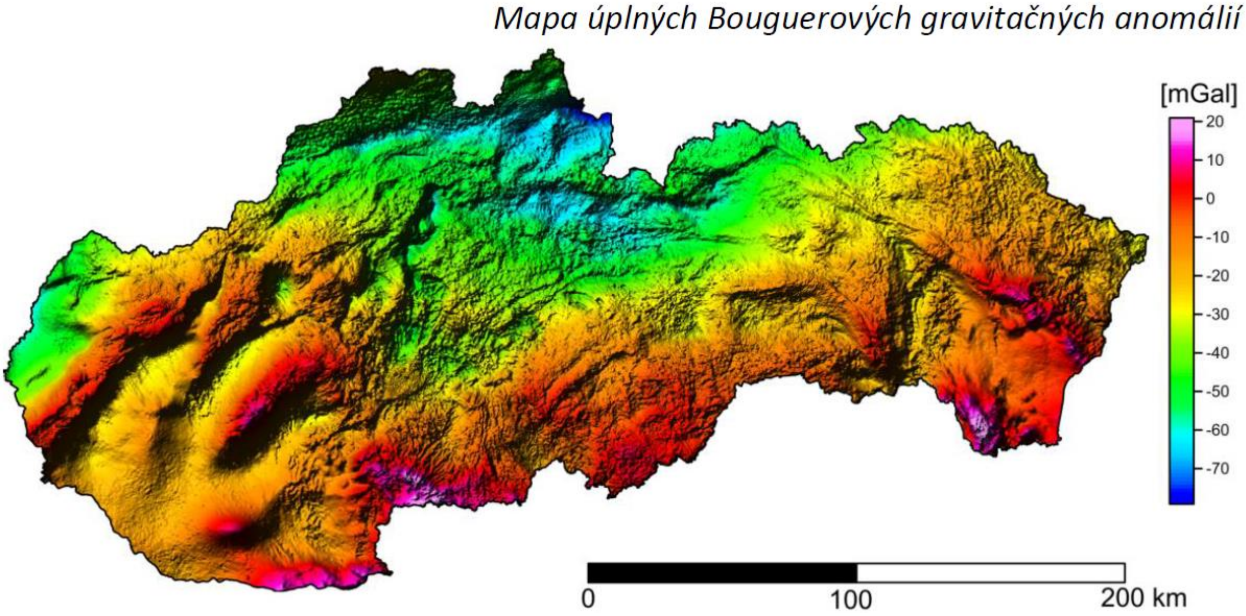
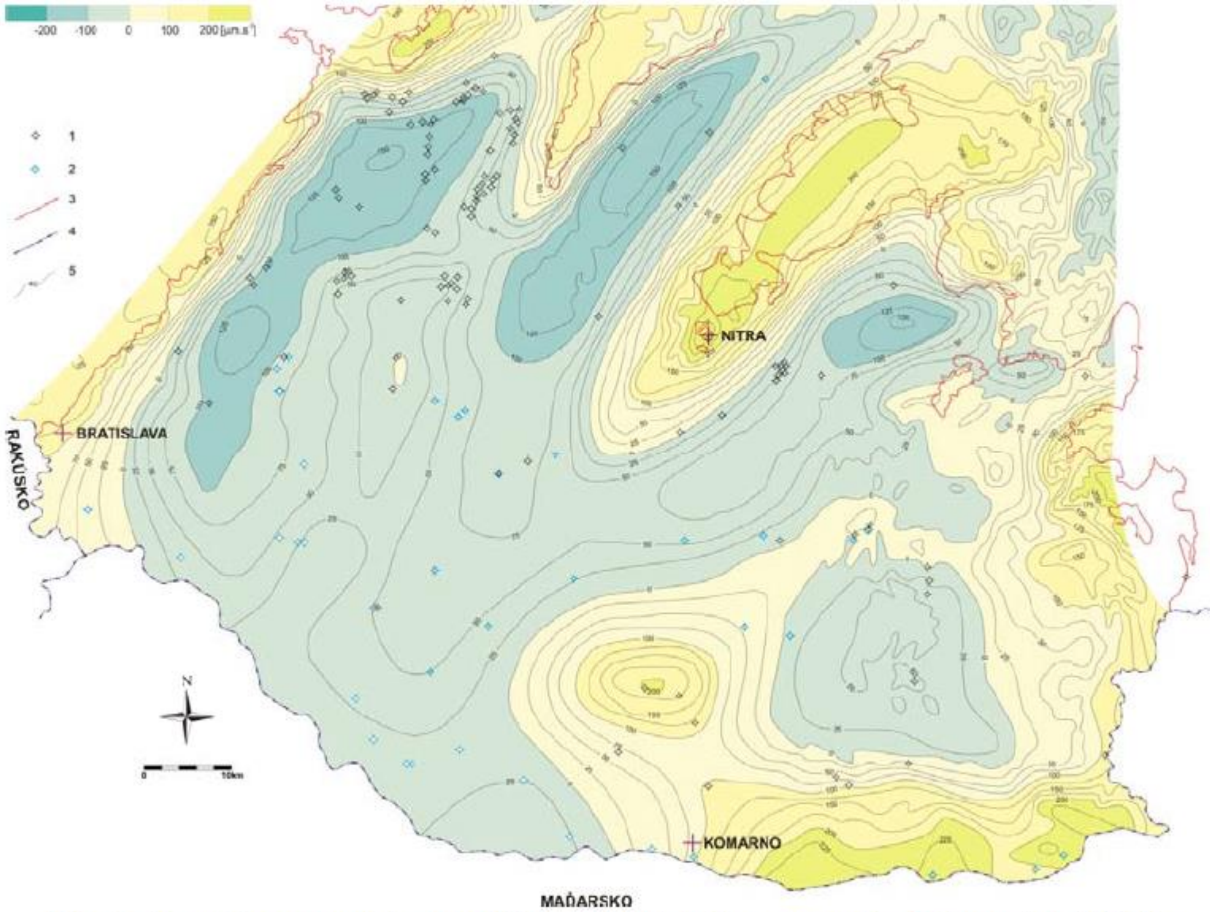


Fig. 3. Map of full Bouguer's anomalies of acceleration of gravity in Slovakia.



Obr. 4: Mapa Bougerových anomálií v podunajskej panve podľa Šefara et al. (1987).

Legenda k obr. 4:

1-vrty uhľovodíkového prieskumu 2-geotermálne vrty, 3-hranica panvy, 4-štátna hranica, 5-izolínie gravitačných Bougerových anomálií v μms^{-2}.

Fig. 4. Map of full Bouguer's anomalies of acceleration of gravity in the Danube lowland in Slovakia.

Tab. 3 Estimated values of full Bouguer's anomalies of acceleration of gravity in the Danube lowland for points of the 1st order levelling network.

Point ID	Location of the point	Estimated value of Bouger´s anomaly (mGal)
CZA-520	Bratislava I – Staré Mesto, Brnianska 31	$\Delta g_{CZA\text{-}520} = (g-\gamma)_B = 12.5$
CZA-544	Bratislava IV – Lamač, Hodonínska 16	$\Delta g_{CZA\text{-}544} = (g-\gamma)_B = 12.5$
CZA-662.01	Veľké Leváre, district Malacky	$\Delta g_{CZA\text{-}662.01} = (g-\gamma)_B = -40.0$
CZA-673	Moravský Svätý Ján, district Senica	$\Delta g_{CZA\text{-}673} = (g-\gamma)_B = -40.0$
ZBZC-544	Sekule – Brodské, state border	$\Delta g_{ZBZC\text{-}544} = (g-\gamma)_B = -40.0$

After estimation of full Bouguer's anomalies of acceleration of gravity on the levelling points of the 1st order levelling network, it was possible to compute the Fay´s anomalies of acceleration of gravity in free air for all 5 points of the 1st order levelling network according to the equation (12). These values are presented in the Tab. 4.

Tab. 4 Fay´s anomalies of acceleration of gravity in free air on the points of the 1st order levelling network.

Point ID	H_{Bpv} (m)	Fay´s anomalies of acceleration of gravity in free air (mGal)
CZA-520	188.6289	$\Delta g_{F(CZA\text{-}520)} = (g-\gamma)_F = (g-\gamma)_B + 0.1119H = 33.608$
CZA-544	217.8081	$\Delta g_{F(CZA\text{-}544)} = (g-\gamma)_F = (g-\gamma)_B + 0.1119H = 36.873$
CZA-662.01	151.3466	$\Delta g_{F(CZA\text{-}662.01)} = (g-\gamma)_F = (g-\gamma)_B + 0.1119H = -23.064$
CZA-673	152.1968	$\Delta g_{F(CZA\text{-}673)} = (g-\gamma)_F = (g-\gamma)_B + 0.1119H = -22.969$
ZBZC-544	154.1453	$\Delta g_{F(ZBZC\text{-}544)} = (g-\gamma)_F = (g-\gamma)_B + 0.1119H = -22.751$

After this step, according to the equation (11), we were able to calculate the earth´s field gravity correction C_{CD}^2 between two points (in the equation (11) symbolically labelled as C and D). The results of this calculation are demonstrated in the Tab. 5, where symbolical labelling C and D in the equation (11) was replaced by concrete point ID´s of the 1st column.

Tab. 5 Computation of earth´s field gravity corrections C_{CD}^{2} between levelling points of the 1st order.

Segments between two points	Levelling height differences between points $\Delta H_{Bpv(CD)} = \Delta h_{CD}^{mer}$ (m)	Mean value of Fay´s anomalies between two points $\Delta g_{F(CD)} = (g-\gamma)_m$ (mGal)	Earth´s field gravity correction C_{CD}^{2} between two points $C_{CD}^{2} = 0.0010193\,(g-\gamma)_m \Delta h_{CD}^{mer}$ (mm)
CZA-544 - CZA-520	29.1792	35.240	**1.05**
CZA-662.01 - CZA-544	-33.2308	6.904	**-0.23**
CZA-673 - CZA-662.01	0.8503	-23.017	**-0.02**
ZBZC-544 - CZA-673	1.9484	-22.860	**-0.05**
Sum of all earth´s field gravity corrections C^2 on the whole segment:			**0.75**

From the computed values of earth´s field gravity corrections for the height differences between concrete pair of the 1st order levelling points (Tab. 4, column 4) and after taking into account the equation (14) is clear that their impact on the individual height differences (Tab 1. column 3) determined in 1951 is insignificant and we can label these differences between two vertical reference frames **Jadran$_{\check{C}SJNS}$ (1951)** a **Bpv (1951)** in the border area between Slovakia and Austria as minor.

Similarly, we also can adopt the same assumption for the differences between **Jadran$_{\check{C}SJNS}$ (1951)** a **Bpv (1951)** on the levelling points of the 2nd order (Tab. 2, column 3).

So, it is unambiguously possible to state that both levelling lines (levelling line of the 1st order - see Tab 1. and levelling line of the 2nd order – see Tab. 2) have reliably determined the values of their mean sea level differences between vertical reference frames ***Jadran$_{\check{C}SJNS}$ (1951)*** *a* ***Bpv (1951)*** *on their levelling points.*

This data set (levelling points of the 1st and 2nd order with their known mean sea level height differences between **Jadran$_{\check{C}SJNS}$ (1951)** a **Bpv (1951)**) serves as an input data set for the interpolation of values of mean sea level height differences between **Jadran$_{\check{C}SJNS}$ (1951)** a **Bpv (1951)** vertical reference frames for all individual points each other 250 m away and geographically located in the middle of Morava and Danube river (they all also have horizontal S-JTSK coordinates). Starting point is in the confluence of Dyje and Morava river, then continuing in the middle of the border river Morava, up to the confluence of rivers Morava and Danube at the village of Devín near Bratislava and then continuing in the middle of Danube river up to the Lafranconi bridge in Bratislava.

Their mean sea level height differences were determined by Kriging´s interpolation method in the whole area of interest. The results are graphically illustrated in the Fig. 5.

In a very few locations (2 or 3 locations), where the Kriging´s method could not be applied, the mean sea level height differences between vertical reference frames **Jadran$_{\textbf{ČSJNS}}$ (1951)** and **Bpv (1951)** were determined by linear interpolation of the nearby points, which still had the mean sea level height differences determined by Kriging´s method.

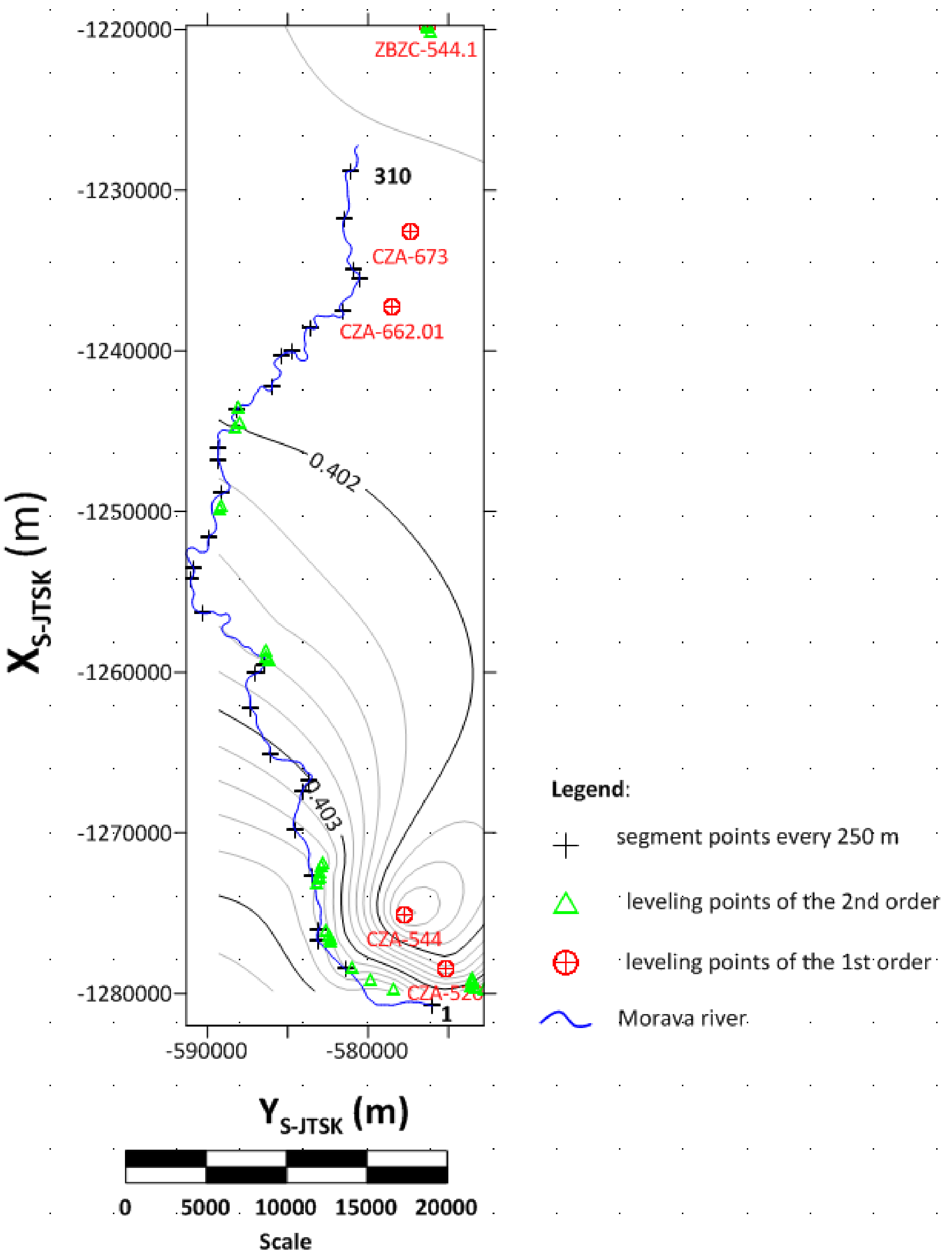


*Fig. 5. Interpolated mean sea level height differences between **Jadran$_{ČSJNS}$ (1951)** a **Bpv (1951)** on the rivers Morava and Danube up to Lafranconi bridge in Bratislava (total segment length: 78.5 km).*

On the Fig. 5 is pictured a small subset of the total amount of 315 points for demonstration purposes only. Final values of the mean sea level height differences between vertical reference frames **Jadran$_{ČSJNS}$** (**1951**) and **Bpv (1951)** for the whole segment of the Morava river (78.5 km long with 315 points each other 250 m away) along with their horizontal S-JTSK coordinates are published in the Technical report (*Mojzeš, Kalafut, 2018*) created for the needs of Slovak Water Management Enterprise, state company, subsidiary Bratislava. Their values oscillate around 40 cm with 2 mm deviation. The lowest value is 0.4018 m and the highest value is 0.4038 m. These values were used for transformation between both vertical frames **in the past**.

The accuracy of the determination of the mean sea level height differences can be estimated by means of the law of error accumulation, where for both vertical reference frames is taken into account the mean error after adjustment of the levelling network, which is lesser than 10 mm (*Vykutil, 1978*):

$$\sigma_{\Delta H} = \sqrt{(\sigma^2_{H^A} + \sigma^2_{H^B})} = 14\,\mathrm{mm}$$

The need for the vertical transformation between two different vertical reference frames (in our case **Jadran$_{ČSJNS}$ (1951) a Bpv (1951))** can also arise in another locations in Slovakia (like the area of oil refinery SLOVNAFT in Bratislava). From scientific point of view, it would be suitable and valuable to determine the mean sea level height differences between both vertical reference frames in the whole levelling network in Slovakia. This would prevent possible errors in their practical applications.

4. Computation of the MSL height differences $\Delta H_{Adria\text{-}Bpv}$ between currently valid Austrian Adriatic vertical reference frame (System der Normalorthometrischen Höhen mit Pegel Adria Triest 1875) and currently valid Slovakian vertical reference frame Bpv (Balt po vyrovnaní) in the middle of border river Morava

The computation of the MSL height differences $\Delta H_{Adria\text{-}Bpv} = H_{Adria} - H_{Bpv}$ (further in text referred to as $\Delta H_{Adria\text{-}Bpv}$ only) between currently valid vertical reference frames in Austria (**System der Normalorthometrischen Höhen mit Pegel Adria Triest 1875**, further in text referred to as **Adria** only) and Slovakia (**Balt po vyrovnaní – Bpv**, further in text referred to as **Bpv**) was realized in the same area of interest like described in the Chapter 3 (in the middle of the border river Morava on the segment about 78.5 km long, with 315 points, each other 250 m away).

The method of the computation was analogical like the one described in the Chapter 3, however, with one important distinction: in contrast to the computation described in the Chapter 3, where were exclusively used Slovakian levelling points only, with their historical MSL heights in both vertical reference frames (Adria and Bpv), here we used not only Slovakian levelling points with their currently valid MSL heights in Slovakian vertical reference frame Bpv, located on the Slovakian side of the border river Morava, but we also used the Austrian levelling points with their currently valid MSL heights in Austrian vertical reference frame (Adria). They are located on the Austrian side of the border river Morava.

Moreover, it is also necessary to mention the following important facts (analogical assertions are also valid for the calculations described in the Chapters 5 and 6):

- Slovakian levelling points have their MSL heights directly determined by the method of precise levelling in the Slovakian vertical reference frame Bpv (H_{Bpv}), however, they do *not* have their MSL heights directly determined by the method of precise levelling in the Austrian vertical reference frame Adria (H_{Adria})
- Austrian levelling points have their MSL heights directly determined by the method of precise levelling in the Austrian vertical reference frame Adria (H_{Adria}), however, they do *not* have their MSL heights directly determined by the method of precise levelling in the Slovakian vertical reference frame Bpv (H_{Bpv})

In order to be able to determine the MSL heights differences $\Delta H_{Adria\text{-}Bpv}$ on these Slovakian and Austrian levelling points, which then would serve as input data for the Kriging´s interpolation method for the determination of the MSL height differences $\Delta H_{Adria\text{-}Bpv}$ on the set of 315 points located in the middle of the border river Morava, it was absolutely necessary to determine their MSL heights in the other vertical reference frame. It means, on the Slovakian levelling points was necessary to determine the MSL (normal orthometric) heights in the Austrian Adriatic vertical reference frame H_{Adria} and on the Austrian levelling points was necessary to determine the MSL (normal Molodensky´s) heights in the Slovakian Baltic vertical reference frame H_{Bpv}. The practical use of precise levelling method was not real, first of all due to economical, but also due to time reasons. As an alternative, we decided to utilize the existence of digital models of Slovakian quasi-geoid (providing normal Molodensky´s

MSL heights in conjunction with global ellipsoidal heights) and of Austrian geoid (providing normal orthometric MSL heights in conjunction with global ellipsoidal heights). Both these (quasi)geoid models have sufficient overlap to the neighbouring country in the area of interest (border river Morava).

The procedure of the computation was as follows:

- from the input horizontal and vertical geodata of Slovakian levelling points ($\boldsymbol{X_{S\text{-}JTSK}}$, $\boldsymbol{Y_{S\text{-}JTSK}}$, $\boldsymbol{H_{Bpv}}$), which have their MSL (normal Molodensky´s) heights determined by direct observations of the precise levelling method in the Slovakian Baltic vertical reference frame Bpv, were determined their geocentric (global) horizontal coordinates $\boldsymbol{\phi_{ETRS\text{-}89}}$, $\boldsymbol{\lambda_{ETRS\text{-}89}}$ and geocentric ellipsoidal heights $\boldsymbol{h_{GRS\text{-}80}}$ by using 3D (three-dimensional) 7-parameter transformation utilizing Slovakian quasi-geoid model,
- from the input horizontal and vertical geodata of Austrian levelling points ($\boldsymbol{X_{GK\_M\text{-}34}}$, $\boldsymbol{Y_{GK\_M\text{-}34}}$, $\boldsymbol{H_{Adria}}$), which have their MSL (normal orthometric) heights determined by direct observations of the precise levelling method in the Austrian Adriatic vertical reference frame Adria, were determined their geocentric (global) horizontal coordinates $\boldsymbol{\phi_{ETRS\text{-}89}}$, $\boldsymbol{\lambda_{ETRS\text{-}89}}$ and geocentric ellipsoidal heights $\boldsymbol{h_{GRS\text{-}80}}$ by using 3D (three-dimensional) 7-parameter transformation utilizing Austrian geoid model.

By these transformations, we achieved that all Austrian and Slovakian levelling points on both sides of Morava river, had determined their geocentric ellipsoidal heights $\boldsymbol{h_{GRS\text{-}80}}$. Hence, it was possible to determine the normal orthometric MSL heights in currently valid Austrian vertical reference frame Adria on Slovakian levelling points utilizing the Austrian geoid model. Similarly, it was possible to determine the normal Molodensky´s MSL heights in currently valid Slovakian Baltic vertical reference frame Bpv on Austrian levelling points utilizing Slovakian quasi-geoid model. It practically means that on the whole set of Austrian and Slovakian levelling points was possible to determine their MSL height differences $\Delta H_{Adria\text{-}Bpv}$.

As input data for the determination of MSL height differences $\Delta H_{Adria\text{-}Bpv}$ on Austrian and Slovakian levelling points were used:

- 57 Austrian levelling points with their currently valid MSL normal orthometric heights in Austrian Adriatic vertical reference frame Adria determined by precise levelling and located on the Austrian side of Morava river,
- 52 Slovakian levelling points with their currently valid MSL normal Molodensky´s heights in Slovakian Baltic vertical reference frame Bpv determined by precise levelling and located on the Slovakian side of Morava river.

The geodata of 57 Austrian levelling points provided Austrian Federal Office for Calibration and Surveying in Vienna, Austria (Bundesamt fur Eich- und Vermessungswessen Wien, www.bev.gv.at) and they can be seen in the Tab. 6. The geodata of 52 Slovakian levelling points were provided by Geodetical and Cartographical Institute, Bratislava, Slovakia (Geodetický a kartografický ústav Bratislava, www.gku.sk) and part of them was taken directly from the geoportal ZBGIS (https://zbgis.skgeodesy.sk) and are also declared in the Tab. 6.

It is important to say on this place that also on the Slovakian side was chosen the same number of levelling points (57) as on Austrian side. The reason for exclusion of 5 Slovakian levelling points (4424BA-1021, 4424BA-1024, C8-516, CZA-610 a CZC-531 labelled with an asterisk (*) in the Tab. 6) was the fact that in the Austrian geoid model was impossible to perform a reliably interpolation in the location of above mentioned Slovakian levelling points (they were on the edge of the area covered by the Austrian geoid model).

Tab. 6 Geodata of Austrian and Slovakian levelling points.

Austrian levelling points – 57 points with approximate horizontal coordinates and currently valid precise normal orthometric MSL heights (Adria) determined by precise levelling				**Slovakian levelling points – 57 points with approximate horizontal coordinates and currently valid normal Molodensy´s MSL heights (Bpv) determined by precise levelling**			
Point ID	**$Y_{GK\_M\text{-}34}$ (m)**	**$X_{GK\_M\text{-}34}$ (m)**	**H_{Adria} (m)**	**Point ID**	**$X_{S\text{-}JTSK}$ (m)**	**$Y_{S\text{-}JTSK}$ (m)**	**H_{Bpv} (m)**
41209A	37 866.00	5 367 023.00	173.805	3441MA-53	1 227 102.07	580 246.75	154.567
41210A	37 939.80	5 367 507.50	175.994	3443MA-42	1 252 510.72	591 177.65	148.694
41211A	37 952.00	5 368 078.00	167.626	4422BA-1002	1 274 474.45	579 239.59	256.721
41218	38 915.12	5 370 180.92	150.437	AZR-529	1 280 618.80	569 354.32	134.342
41219	39 071.09	5 370 285.23	149.229	C59-509	1 230 730.16	571 396.88	173.259
41220A	39 202.00	5 369 893.00	149.393	CZA-501	1 279 759.34	572 785.25	141.523
41221A	39 324.00	5 369 811.00	146.128	CZA-503	1 279 656.07	573 295.73	152.761
41232A	40 421.90	5 373 219.10	153.014	CZA-505	1 279 603.79	573 585.90	159.677
41233	40 866.41	5 373 071.43	150.755	CZA-506	1 279 381.32	573 568.09	162.813
41236	40 257.43	5 373 731.17	158.419	CZA-508	1 279 260.68	573 504.27	166.283
41253	42 140.36	5 377 893.23	154.971	CZA-509	1 279 169.67	573 547.03	176.052
41256	42 322.47	5 377 871.76	153.288	CZA-520	1 278 448.87	575 144.40	188.629
41285A	43 455.60	5 384 935.00	153.318	CZA-544	1 275 092.81	577 690.00	217.808
41285B	44 153.63	5 385 118.02	153.532	CZA-644	1 242 166.13	576 971.78	155.882
41285C	44 316.50	5 384	151.086	CZA-	1 237 279.37	578 462.14	151.346

		941.20		662.01			
41285D	44 319.50	5 384 928.70	151.213	CZA-673	1 232 601.36	577 324.40	152.197
41285E	44 320.00	5 384 848.00	151.829	ZA1-505	1 279 829.77	578 371.14	140.722
41184A	37 229.00	5 360 807.20	144.824	ZA1-510	1 279 212.53	579 805.55	143.138
41184B	37 267.49	5 360 755.07	144.072	ZA1-516	1 278 457.07	581 033.46	152.522
41184C	37 163.00	5 360 653.30	144.333	ZA1-521	1 276 818.83	582 349.61	147.088
41184D	37 160.20	5 360 658.80	144.918	ZA1-522	1 276 659.97	582 329.15	157.812
41184E	37 171.00	5 360 648.00	141.111	ZA1-525	1 276 472.47	582 464.35	152.719
41095C	44 914.40	5 345 028.90	148.390	ZA1-526	1 276 183.70	582 626.21	149.441
41096	45 410.20	5 344 958.30	150.358	ZA1-537	1 273 182.86	583 167.77	142.248
41096A	45 175.70	5 344 975.80	150.133	ZA1-539	1 272 905.53	583 134.29	145.562
41097	45 360.90	5 344 946.00	145.920	ZA2-500	1 272 776.10	583 008.26	149.718
41097A	45 378.20	5 344 945.90	144.253	ZA2-501	1 272 512.63	582 970.64	150.977
41097	45 506.80	5 344 946.95	141.232	ZA2-502	1 272 181.68	582 866.73	151.913
41099	45 557.90	5 344 985.00	140.224	ZA2-503	1 271 952.21	582 803.78	143.447
41102	45 598.10	5 344 950.10	150.630	ZA5-505	1 273 262.42	580 822.62	163.994
41103	45 599.50	5 344 941.90	150.622	ZA10-517	1 259 252.89	586 200.15	145.335
41074	46 930.40	5 339 325.20	144.704	ZA10-518	1 259 080.03	586 324.41	144.622
41075	46 856.86	5 339 414.84	141.437	ZA10-520	1 258 685.29	586 408.11	145.084
41075B	46 648.46	5 339 476.12	144.603	ZA18-511.02	1 243 611.38	588 055.69	145.445
41076	46 324.37	5 339 646.66	143.730	ZA18-513	1 244 552.74	588 021.15	145.728
38840C	46 174.39	5 335 628.68	197.460	ZA18-513.02	1 244 870.79	588 271.87	145.622
38840D	46 495.60	5 335	224.317	ZA18-530	1 249 733.66	589 184.73	146.412

		785.80					
38840E	46 605.59	5 335 453.05	285.792	ZA18-532	1 249 883.44	589 261.04	145.316
38849	47 902.80	5 334 089.10	161.604	ZA18-537	1 251 644.28	589 388.25	143.966
38856	49 795.00	5 333 494.70	148.519	ZA19-509	1 251 583.89	586 310.29	146.381
38867	51 495.00	5 332 494.99	141.247	ZA21-507	1 244 602.79	582 269.87	150.091
38868D	53 818.80	5 331 729.40	141.688	ZA23-505	1 240 779.73	578 765.43	152.865
41138A	40 138.23	5 352 720.15	144.688	ZA23-511	1 242 413.49	580 750.82	151.106
41138B	40 327.70	5 353 055.60	143.294	ZAZB-520	1 229 017.49	581 356.84	150.244
41138C	39 979.80	5 353 247.40	143.672	ZBZC-509	1 226 781.94	576 932.10	152.050
41138D	39 974.76	5 353 713.40	141.870	ZBZC-539	1 220 244.48	576 105.64	155.023
41201	37 912.83	5 363 771.23	149.856	ZBZC-540	1 219 901.90	576 293.36	154.667
41202	38 063.00	5 363 810.37	149.130	ZBZC-541	1 219 830.29	576 331.94	154.682
41203	38 063.02	5 363 811.00	150.748	ZBZC-544	1 219 729.46	576 333.79	154.145
41268	39 883.80	5 380 345.67	162.033	ZC7-513	1 218 886.94	572 948.74	161.650
41269	40 020.55	5 380 401.75	157.634	ZNSBA2-507	1 287 533.95	577 099.13	134.443
41270	40 570.35	5 380 659.10	153.622	ZNSBA11-502	1 282 500.93	569 173.24	135.438
41271A	41 054.70	5 381 112.35	155.754	4424BA-1021*	1 290 550.66	562 356.95	128.688
41299	42 162.40	5 390 293.60	168.543	4424BA-1024*	1 294 172.28	566 034.08	134.707
41300	42 162.43	5 390 293.64	170.040	C8-516*	1 258 610.34	573 389.83	212.189
41301	42 069.50	5 390 442.65	170.412	CZA-610*	1 254 610.75	577 682.49	167.175
41302	41 916.99	5 390 345.39	168.568	CZC-531*	1 226 422.39	567 555.30	167.166

As the first step, it was necessary to determine the MSL height differences $\Delta H_{Adria\text{-}Bpv}$ for all 109 Slovakian and Austrian levelling points. These differences on all 109 levelling points became the input values for the determination of the MSL height differences $\Delta H_{Adria\text{-}Bpv}$ for all 315 points (each other 250 m away) in the area of interest (in the middle of Morava river, about 78.5 km long segment). The MSL height differences $\Delta H_{Adria\text{-}Bpv}$ for all 109 levelling points were determined

- by using Slovakian quasi-geoid model DVRM05, ver. 05.2005 with its height (undulation) accuracy of about 3 cm (the mean value of residuals – 1σ on testing points is 34 mm, see https://www.geoportal.sk/sk/geodeticke-zaklady/geodeticke-systemy-transformacie/), which serves for height transformation of global ellipsoidal heights related to geocentric GRS80 ellipsoid onto normal Molodensky´s heights in Slovakian Baltic vertical reference frame Bpv,
  by using Austrian geoid model with its height (undulation) accuracy of about 3 cm (the mean value of residuals – 1σ on testing points is 35 mm, see http://www.bev.gv.at/portal/page?_pageid=713,2363173&_dad=portal&_schema=PORTAL), which serves for height transformation of global ellipsoidal heights related to geocentric GRS80 ellipsoid onto normal orthometric heights in Austrian Adriatic vertical reference frame Adria.

Of course, both (quasi)geoids must have sufficient overlap to neighbouring countries (at least several km). Both mentioned (quasi)geoid models comply to this requirement. It is also important to state that both models (Austrian geoid and Slovakian quasi-geoid) contain mean tide model.

Schematic coverage of the territory by Slovakian quasi-geoid is illustrated in the Fig. 6. (see the rectangle bordered by black lines). Slovakian quasi-geoid covers the territory in the direction from the south to the north between latitudes of N47°30´10.00800" and N49°59´49.99200" and in the direction from the west to the east between longitudes E16°30´15.01200" and E22°59´44.98800".

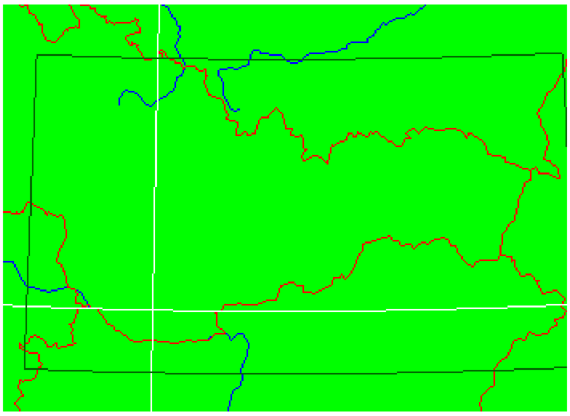


Fig. 6. Geographical extent of Slovakian quasi-geoid DVRM05, ver. 05.2005.

Schematic coverage of the territory by Austrian geoid is illustrated in the Fig. 7 (see the rectangle bordered by black lines). Austrian geoid covers the territory in the direction from the south to the north between latitudes of N46°20´30.00000“and N49°03´15.00000“ and in the direction form the west to the east between longitudes E9°28´52.50000“ a E17°12´22.50000“.

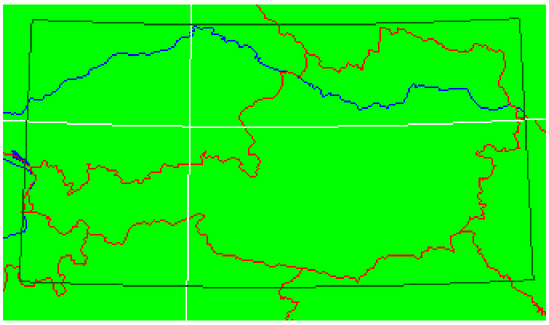


Fig. 7. Geographical extent of Austrian geoid.

The overlap of both models in the border river Morava in the south – north direction corresponds to the value of 1°33´04.992“(approximately 172.5 km) and in the west – east direction corresponds to the value of 0°42´07.488“(approximately 50.3 km). This overlap is fully enough for our purposes of the determination of MSL height differences $\Delta H_{Adria\text{-}Bpv}$ in the border area between Austria and Slovakia.

Although the horizontal absolute accuracy of Austrian and Slovakian levelling points is quite low and generally is on the level of 1 – 5 m (except a few exceptions, first of all on Slovakian levelling points, where in some cases, the horizontal accuracy is on the level of a few cm), this deficit of horizontal accuracy has practically no real negative impact for the determination of MSL height differences $\Delta H_{Adria\text{-}Bpv}$ between vertical reference frames Adria and Bpv, particularly when we take into account the following key facts:

- the whole nearby area of border river Morava on Austrian – Slovakian border is practically a plane,
- the step of discrete nodes of the grid of Slovakian quasi-geoid model, in which the quadratic interpolation is carried out for the determination of quasi-geoid heights (undulations) for the points outside the grid nodes is about 600 m x 600 m,
- the step of discrete nodes of the grid of Austrian geoid model, in which the quadratic interpolation is carried out for the determination of geoid heights (undulations) for the points outside the grid nodes is about 500 m x 500 m.

In the Tab. 7 are illustrated the MSL height differences $\Delta H_{Adria\text{-}Bpv}$ on Austrian and Slovakian levelling points determined by the method described above. To repeat it in short, the MSL heights in the Slovakian Baltic vertical reference frame Bpv on Austrian levelling points were obtained by using Slovakian quasi-geoid model and the MSL heights in the Austrian Adriatic vertical reference frame on Slovakian levelling points were obtained by using Austrian geoid model.

The MSL height differences $\Delta H_{Adria\text{-}Bpv}$ on Austrian levelling points lie in the interval of 0.526 m – 0.563 m (their spread is 0.037 m) and on Slovakian levelling points lie in the interval of 0.357 m – 0.557 m (their spread is 0.200 m). Quite large (20 cm) spread of the $\Delta H_{Adria\text{-}Bpv}$ values on Slovakian levelling points is caused by two significant MSL height differences $\Delta H_{Adria\text{-}Bpv}$ on the point AZR-529 (0.357 m) and the point ZNSBA11-502 (0.358 m). Without these two points, the spread would be 0.097 m. As a possible explanation of this fact (quite large spread of $\Delta H_{Adria\text{-}Bpv}$ differences on Slovakian levelling points) can be that their normal (Molodensky´s) heights were not homogeneously determined. Moreover, both points AZR-529 and ZNSBA11-502 need to be verified by a new precise levelling.

Tab. 7 MSL height differences $\Delta H_{Adria\text{-}Bpv}$ on Austrian and Slovakian levelling points.

Austrian levelling points – 57 points				**Slovakian levelling points – 52 points**			
Point ID	H_{Adria} **(m)**	H_{Bpv} **(m)**	$\Delta H_{Adria\text{-}Bpv}$ **(m)**	**Point ID**	H_{Adria} **(m)**	H_{Bpv} **(m)**	$\Delta H_{Adria\text{-}Bpv}$ **(m)**
38840C	197.460	196.903	**0.557**	3441MA-53	155.115	154.567	**0.548**
38840D	224.317	223.760	**0.557**	3443MA-42	149.246	148.694	**0.552**
38840E	285.792	285.238	**0.554**	4422BA-1002	257.278	256.721	**0.557**
38849	161.604	161.059	**0.545**	AZR-529	134.699	134.342	**0.357**
38856	148.519	147.978	**0.541**	C59-509	173.812	173.259	**0.553**
38867	141.247	140.711	**0.536**	CZA-501	141.983	141.523	**0.460**
38868D	141.688	141.154	**0.534**	CZA-503	153.235	152.761	**0.474**
41074	144.704	144.143	**0.561**	CZA-505	160.159	159.677	**0.482**
41075	141.437	140.876	**0.561**	CZA-506	163.297	162.813	**0.484**
41075B	144.603	144.042	**0.561**	CZA-508	166.765	166.283	**0.482**
41076	143.730	143.167	**0.563**	CZA-509	176.537	176.052	**0.485**
41095C	148.390	147.840	**0.550**	CZA-520	189.146	188.629	**0.517**
41096	150.358	149.810	**0.548**	CZA-544	218.347	217.808	**0.539**
41096A	150.133	149.585	**0.548**	CZA-644	156.399	155.882	**0.517**
41097	145.920	145.372	**0.548**	CZA-662.01	151.858	151.346	**0.512**
41097A	144.253	143.705	**0.548**	CZA-673	152.707	152.197	**0.510**
41098	141.232	140.685	**0.547**	ZA1-505	141.260	140.722	**0.538**
41099	140.224	139.678	**0.546**	ZA1-510	143.678	143.138	**0.540**
41102	150.630	150.083	**0.547**	ZA1-516	153.066	152.522	**0.544**
41103	150.622	150.075	**0.547**	ZA1-521	147.638	147.088	**0.550**
41138A	144.688	144.140	**0.548**	ZA1-522	158.362	157.812	**0.550**
41138B	143.294	142.747	**0.547**	ZA1-525	153.270	152.719	**0.551**
41138C	143.672	143.123	**0.549**	ZA1-526	149.994	149.441	**0.553**
41138D	141.870	141.322	**0.548**	ZA1-537	142.801	142.248	**0.553**
41184A	144.824	144.274	**0.550**	ZA1-539	146.114	145.562	**0.552**
41184B	144.072	143.522	**0.550**	ZA2-500	150.268	149.718	**0.550**

41184C	144.333	143.782	**0.551**	ZA2-501	151.525	150.977	**0.548**
41184D	144.918	144.367	**0.551**	ZA2-502	152.459	151.913	**0.546**
41184E	141.111	140.560	**0.551**	ZA2-503	143.991	143.447	**0.544**
41201	149.856	149.310	**0.546**	ZA5-505	164.550	163.994	**0.556**
41202	149.130	148.586	**0.544**	ZA10-517	145.877	145.335	**0.542**
41203	150.748	150.204	**0.544**	ZA10-518	145.164	144.622	**0.542**
41209A	173.805	173.253	**0.552**	ZA10-520	145.626	145.084	**0.542**
41210A	175.994	175.445	**0.549**	ZA18-511.02	145.981	145.445	**0.536**
41211A	167.626	167.077	**0.549**	ZA18-513	146.262	145.728	**0.534**
41218	150.437	149.896	**0.541**	ZA18-513.02	146.158	145.622	**0.536**
41219	149.229	148.692	**0.537**	ZA18-530	146.955	146.412	**0.543**
41220A	149.393	148.854	**0.539**	ZA18-532	145.860	145.316	**0.544**
41221A	146.128	145.590	**0.538**	ZA18-537	144.513	143.966	**0.547**
41232A	153.014	152.477	**0.537**	ZA19-509	146.893	146.381	**0.512**
41233	150.755	150.223	**0.532**	ZA21-507	150.594	150.091	**0.503**
41236	158.419	157.884	**0.535**	ZA23-505	153.357	152.865	**0.492**
41253	154.971	154.444	**0.527**	ZA23-511	151.626	151.106	**0.520**
41256	153.288	152.762	**0.526**	ZAZB-520	150.790	150.244	**0.546**
41268	162.033	161.486	**0.547**	ZBZC-509	152.572	152.050	**0.522**
41269	157.634	157.086	**0.548**	ZBZC-539	155.512	155.023	**0.489**
41270	153.622	153.077	**0.545**	ZBZC-540	155.153	154.667	**0.486**
41271A	155.754	155.202	**0.552**	ZBZC-541	155.168	154.682	**0.486**
41285A	153.318	152.768	**0.550**	ZBZC-544	154.630	154.145	**0.485**
41285B	153.532	152.985	**0.547**	ZC7-513	162.124	161.650	**0.474**
41285C	151.086	150.540	**0.546**	ZNSBA2-507	134.984	134.443	**0.541**
41285D	151.213	150.667	**0.546**	ZNSBA11-502	135.796	135.438	**0.358**
41285E	151.829	151.283	**0.546**	4424BA-1021*	Not used	Not used	Not used
41299	168.543	167.984	**0.559**	4424BA-1024*	Not used	Not used	Not used
41300	170.040	169.481	**0.559**	C8-516*	Not used	Not used	Not used
41301	170.412	169.852	**0.560**	CZA-610*	Not used	Not used	Not used
41302	168.568	168.007	**0.561**	CZC-531*	Not used	Not used	Not used

The MSL height differences $\Delta H_{Adria\text{-}Bpv}$ in the Tab. 7 (columns 4 and 8) constitute the input data for the determination of additional MSL height differences $\Delta H_{Adria\text{-}Bpv}$ for all 315 points (each other 250 m away) in the area of interest (located in the middle of the border river Morava). These values were determined by Kriging´s interpolation method and their graphical shape is illustrated in the Fig. 8.

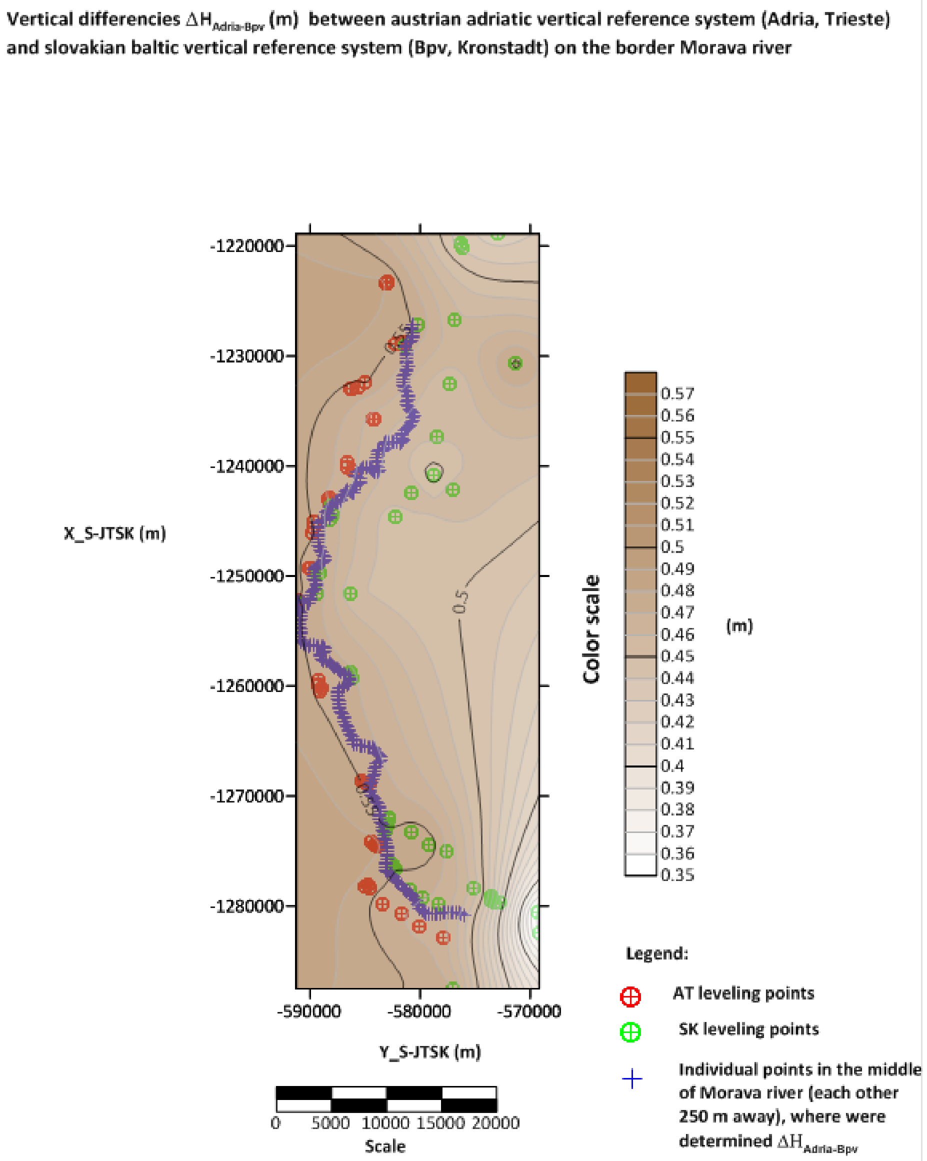
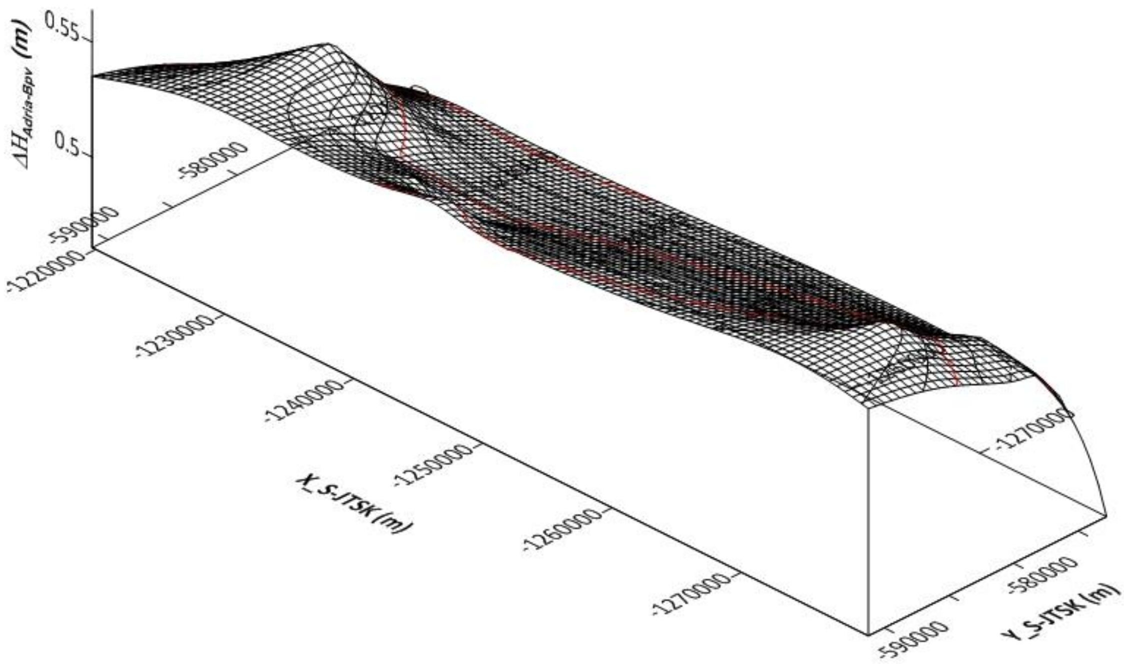


Fig. 8. Vertical differences $\Delta H_{Adria\text{-}Bpv}$ between Austrian Adriatic vertical reference frame (Adria, Terst) and Slovakian Baltic vertical reference frame (Bpv, Kronštadt) on the border river Morava.

Another way of graphical illustration of the vertical differences $\Delta H_{Adria\text{-}Bpv}$ using orthographic projection can be seen in the Fig. 9.



Obr. 9. Vertical differences $\Delta H_{Adria-Bpv}$ between Austrian Adriatic vertical reference frame (Adria, Terst) and Slovakian Baltic vertical reference frame (Bpv, Kronštadt) on the border river Morava in orthographic projection.

The mean value of the MSL height differences $\Delta H_{Adria-Bpv}$ on all Austrian and Slovakian levelling points (together 109 points) is 0.534 m. Positive values of $\Delta H_{Adria-Bpv}$ mean that the initial point in Austrian Adriatic vertical reference frame Adria in Trieste (point with normal orthometric height H_{Adria} = 0 m) is lower with respect to the initial point in Slovakian Baltic vertical reference frame Bpv in Kronstadt (point with normal height H_{Bpv} = 0 m). In the Tab. 8, the residuals on individual levelling points are demonstrated. The root mean square (RMS) error **σ_0 = 33 mm** and mean error of arithmetic average **σ = 3 mm** of vertical differences $\Delta H_{Adria-Bpv}$ was computed from these residuals.

Tab. 8 Residuals of vertical differences $\Delta H_{Adria\text{-}Bpv}$ on Austrian and Slovakian levelling points.

Austrian levelling points – 57 points			**Slovakian levelling points– 52 points**		
Point ID	$\boldsymbol{\Delta H_{Adria\text{-}Bpv}}$ **(m)**	**Residuals (m)**	**Point ID**	$\boldsymbol{\Delta H_{Adria\text{-}Bpv}}$ **(m)**	**Residuals (m)**
38840 C	0.557	-0.023	3441MA-53	0.548	-0.014
38840 D	0.557	-0.023	3443MA-42	0.552	-0.018
38840 E	0.554	-0.020	4422BA-1002	0.557	-0.023
38849	0.545	-0.011	AZR-529	0.357	0.177
38856	0.541	-0.007	C59-509	0.553	-0.019
38867	0.536	-0.002	CZA-501	0.460	0.074
38868 D	0.534	0.000	CZA-503	0.474	0.060
41074	0.561	-0.027	CZA-505	0.482	0.052
41075	0.561	-0.027	CZA-506	0.484	0.050
41075 B	0.561	-0.027	CZA-508	0.482	0.052
41076	0.563	-0.029	CZA-509	0.485	0.049
41095 C	0.550	-0.016	CZA-520	0.517	0.017
41096	0.548	-0.014	CZA-544	0.539	-0.005
41096 A	0.548	-0.014	CZA-644	0.517	0.017
41097	0.548	-0.014	CZA-662.01	0.512	0.022
41097 A	0.548	-0.014	CZA-673	0.510	0.024
41098	0.547	-0.013	ZA1-505	0.538	-0.004
41099	0.546	-0.012	ZA1-510	0.540	-0.006
41102	0.547	-0.013	ZA1-516	0.544	-0.010
41103	0.547	-0.013	ZA1-521	0.550	-0.016
41138 A	0.548	-0.014	ZA1-522	0.550	-0.016
41138 B	0.547	-0.013	ZA1-525	0.551	-0.017
41138 C	0.549	-0.015	ZA1-526	0.553	-0.019
41138 D	0.548	-0.014	ZA1-537	0.553	-0.019
41184	0.550	-0.016	ZA1-539	0.552	-0.018

A					
41184 B	0.550	-0.016	ZA2-500	0.550	-0.016
41184 C	0.551	-0.017	ZA2-501	0.548	-0.014
41184 D	0.551	-0.017	ZA2-502	0.546	-0.012
41184 E	0.551	-0.017	ZA2-503	0.544	-0.010
41201	0.546	-0.012	ZA5-505	0.556	-0.022
41202	0.544	-0.010	ZA10-517	0.542	-0.008
41203	0.544	-0.010	ZA10-518	0.542	-0.008
41209 A	0.552	-0.018	ZA10-520	0.542	-0.008
41210 A	0.549	-0.015	ZA18-511.02	0.536	-0.002
41211 A	0.549	-0.015	ZA18-513	0.534	0.000
41218	0.541	-0.007	ZA18-513.02	0.536	-0.002
41219	0.537	-0.003	ZA18-530	0.543	-0.009
41220 A	0.539	-0.005	ZA18-532	0.544	-0.010
41221 A	0.538	-0.004	ZA18-537	0.547	-0.013
41232 A	0.537	-0.003	ZA19-509	0.512	0.022
41233	0.532	0.002	ZA21-507	0.503	0.031
41236	0.535	-0.001	ZA23-505	0.492	0.042
41253	0.527	0.007	ZA23-511	0.520	0.014
41256	0.526	0.008	ZAZB-520	0.546	-0.012
41268	0.547	-0.013	ZBZC-509	0.522	0.012
41269	0.548	-0.014	ZBZC-539	0.489	0.045
41270	0.545	-0.011	ZBZC-540	0.486	0.048
41271 A	0.552	-0.018	ZBZC-541	0.486	0.048
41285 A	0.550	-0.016	ZBZC-544	0.485	0.049
41285 B	0.547	-0.013	ZC7-513	0.474	0.060
41285 C	0.546	-0.012	ZNSBA2-507	0.541	-0.007
41285 D	0.546	-0.012	ZNSBA11-502	0.358	0.176

41285 E	0.546	-0.012	4424BA-1021*	Not used	Not used
41299	0.559	-0.025	4424BA-1024*	Not used	Not used
41300	0.559	-0.025	C8-516*	Not used	Not used
41301	0.560	-0.026	CZA-610*	Not used	Not used
41302	0.561	-0.027	CZC-531*	Not used	Not used

Exact numerical values of the MSL height differences $\Delta H_{Adria\text{-}Bpv}$ for all 315 points located in the middle of border river Morava are not presented here due to spatial reasons. Their values range in the interval of **0.515 m** – **0.558 m** (their spread is **0.043 m**).

5. Computation of the MSL height differences $\Delta H_{EVRF2007\text{-}Bpv}$ between currently valid Slovakian Baltic vertical reference frame Bpv and European vertical reference frame EVRF2007 in the middle of border river Morava

The computation of MSL height differences $\Delta H_{EVRF2007\text{-}Bpv} = H_{EVRF2007} - H_{Bpv}$ (in further text also referred to as $\Delta H_{EVRF2007\text{-}Bpv}$) was carried out by the same manner as described in the Chapter 4. The only difference in this case was that five additional Slovakian levelling points were included in the computation (4424BA-1021, 4424BA-1024, C8-516, CZA-610 a CZC-531, see Tab. 6), which were previously excluded due to their horizontal positions (they lie on the edge of area covered by Austrian geoid). All calculations were realized in the same area of interest (border river Morava) and for the same amount of 315 individual points located in the middle of the river Morava.

Since in the computations were also included Austrian levelling points, it is necessary to say that for the computation of MSL height differences $\Delta H_{EVRF2007\text{-}Bpv}$ on Austrian levelling points were considered their Bpv MSL heights determined by using Slovakian quasi-geoid model DVRM05, whereas the MSL heights on Slovakian levelling points were MSL heights directly determined by precise levelling method. The reason for inclusion of Austrian levelling points with their MSL heights in Slovakian Baltic vertical reference frame Bpv determined by using Slovakian quasi-geoid model in the computation of the MSL height differences $\Delta H_{EVRF2007\text{-}Bpv}$ was to preserve as much levelling points on which were determined the MSL height differences $\Delta H_{EVRF2007\text{-}Bpv}$, as possible, because they served as input data for the determination of the MSL height differences $\Delta H_{EVRF2007\text{-}Bpv}$ by Kriging´s interpolation method on individual 315 points located in the middle of border river Morava. For the determination of normal (Molodensky´s) MSL heights in the vertical reference frame EVRF2007 was utilized Slovakian quasi-geoid model DMQSK2014-E, version 06.2014. Its height (undulation) accuracy is on the level of 2.3 cm (mean value of residuals 1σ on tested points was 2.3 cm (see https://www.geoportal.sk/sk/geodeticke-zaklady/geodeticke-systemy-transformacie/). Its geographical extent is illustrated in the Fig. 10.

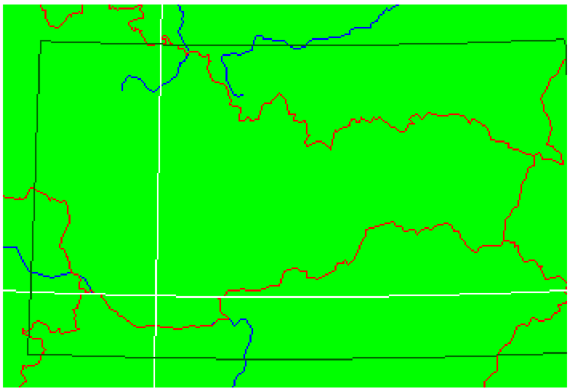


Fig. 10. Geographical extent (bordered by black rectangle) of Slovakian quasi-geoid model DMQSK2014-E, ver. 06.2014 for the computation of normal MSL heights in the vertical reference frame EVRF2007.

This quasi-geoid covers whole territory of Slovakia with overlapping to neighbouring states (see black rectangle in the Fig. 10) and is bordered by latitudes between N47°30´29.98000“ and N49°59´29.98400“ and by longitudes between E16°30´44.98802“ and E22°59´15.01198“.

MSL vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ in the Tab. 9 (4th and 8th column) represent input data for the determination of these differences ($\Delta H_{EVRF2007\text{-}Bpv}$) on 315 individual points (each other 250 m away) by Kriging´s interpolation method and located in the middle of the border river Morava. They graphical presentation can be seen in the Fig. 11.

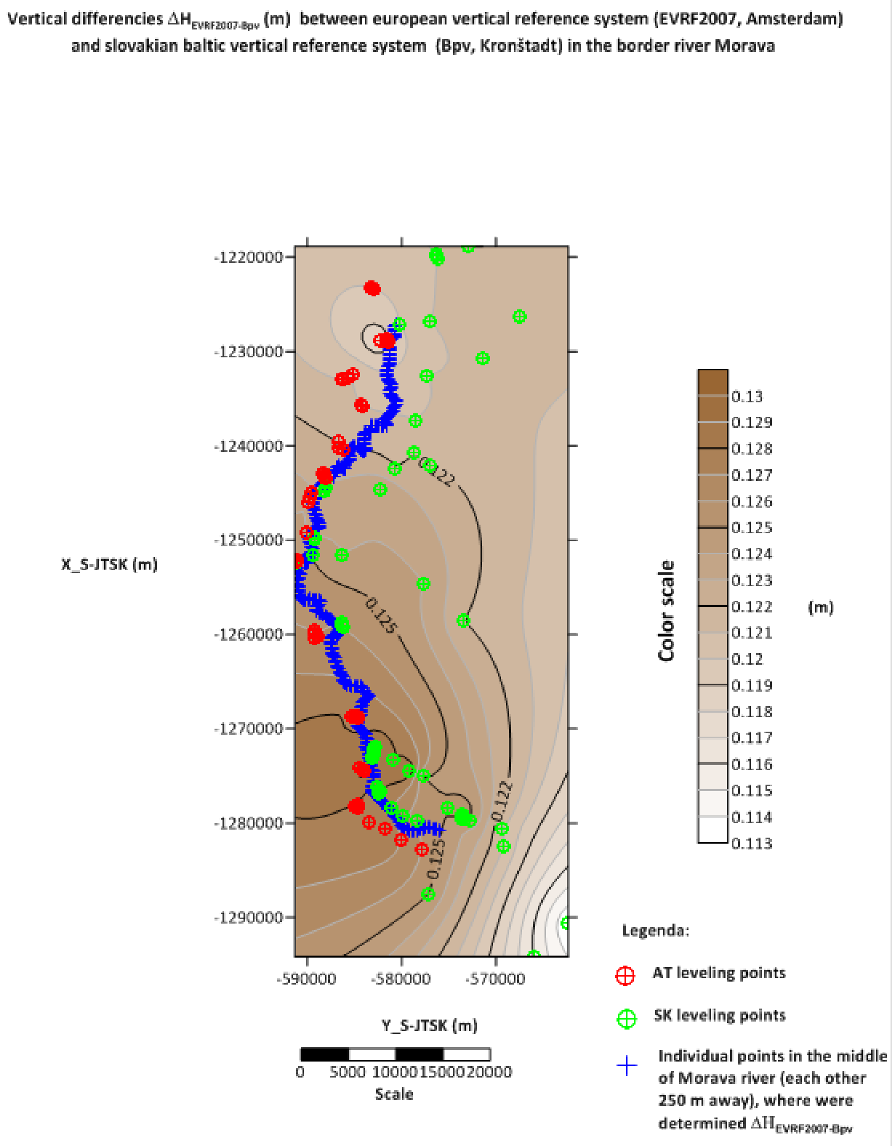


Fig. 11. Vertical differences $\Delta H_{EVRF2007-Bpv)}$ (m) between European vertical reference frame (EVRF2007, Amsterdam) and Slovakian Baltic reference frame (Bpv, Kronštadt) on the border river Morava.

Tab. 9 Vertical MSL differences $\Delta H_{EVRF2007\text{-}Bpv}$ on Austrian and Slovakian levelling points.

Austrian levelling points – 57 points				**Slovakian levelling points – 57 points**			
Point ID	$H_{EVRF2007}$ **(m)**	H_{Bpv} **(m)**	$\Delta H_{EVRF2007\text{-}Bpv}$ **(m)**	**Point ID**	$H_{EVRF2007}$ **(m)**	H_{Bpv} **(m)**	$\Delta H_{EVRF2007\text{-}Bpv}$ **(m)**
38840C	197.031	196.903	**0.128**	3441MA-53	154.687	154.567	**0.120**
38840D	223.888	223.760	**0.128**	3443MA-42	148.819	148.694	**0.125**
38840E	285.366	285.238	**0.128**	4422BA-1002	256.850	256.721	**0.129**
38849	161.186	161.059	**0.127**	AZR-529	134.463	134.342	**0.121**
38856	148.106	147.978	**0.128**	C59-509	173.380	173.259	**0.121**
38867	140.838	140.711	**0.127**	CZA-501	141.648	141.523	**0.125**
38868D	141.280	141.154	**0.126**	CZA-503	152.885	152.761	**0.124**
41074	144.271	144.143	**0.128**	CZA-505	159.801	159.677	**0.124**
41075	141.004	140.876	**0.128**	CZA-506	162.938	162.813	**0.125**
41075B	144.169	144.042	**0.127**	CZA-508	166.408	166.283	**0.125**
41076	143.296	143.167	**0.129**	CZA-509	176.178	176.052	**0.126**
41095C	147.968	147.840	**0.128**	CZA-520	188.754	188.629	**0.125**
41096	149.938	149.810	**0.128**	CZA-544	217.933	217.808	**0.125**
41096A	149.712	149.585	**0.127**	CZA-644	156.004	155.882	**0.122**
41097	145.499	145.372	**0.127**	CZA-662.01	151.467	151.346	**0.121**
41097A	143.832	143.705	**0.127**	CZA-673	152.318	152.197	**0.121**
41098	140.812	140.685	**0.127**	ZA1-505	140.848	140.722	**0.126**
41099	139.805	139.678	**0.127**	ZA1-510	143.264	143.138	**0.126**
41102	150.211	150.083	**0.128**	ZA1-516	152.649	152.522	**0.127**
41103	150.202	150.07	**0.127**	ZA1-521	147.216	147.08	**0.128**

		5				8	
41138A	144.266	144.140	**0.126**	ZA1-522	157.940	157.812	**0.128**
41138B	142.873	142.747	**0.126**	ZA1-525	152.847	152.719	**0.128**
41138C	143.249	143.123	**0.126**	ZA1-526	149.569	149.441	**0.128**
41138D	141.448	141.322	**0.126**	ZA1-537	142.377	142.248	**0.129**
41184A	144.399	144.274	**0.125**	ZA1-539	145.691	145.562	**0.129**
41184B	143.647	143.522	**0.125**	ZA2-500	149.846	149.718	**0.128**
41184C	143.907	143.782	**0.125**	ZA2-501	151.105	150.977	**0.128**
41184D	144.492	144.367	**0.125**	ZA2-502	152.042	151.913	**0.129**
41184E	140.685	140.560	**0.125**	ZA2-503	143.576	143.447	**0.129**
41201	149.434	149.310	**0.124**	ZA5-505	164.122	163.994	**0.128**
41202	148.710	148.586	**0.124**	ZA10-517	145.461	145.335	**0.126**
41203	150.328	150.204	**0.124**	ZA10-518	144.748	144.622	**0.126**
41209A	173.377	173.253	**0.124**	ZA10-520	145.210	145.084	**0.126**
41210A	175.568	175.445	**0.123**	ZA18-511.02	145.568	145.445	**0.123**
41211A	167.200	167.077	**0.123**	ZA18-513	145.851	145.728	**0.123**
41218	150.019	149.896	**0.123**	ZA18-513.02	145.745	145.622	**0.123**
41219	148.815	148.692	**0.123**	ZA18-530	146.536	146.412	**0.124**
41220A	148.977	148.854	**0.123**	ZA18-532	145.440	145.316	**0.124**
41221A	145.713	145.590	**0.123**	ZA18-537	144.091	143.966	**0.125**
41232A	152.599	152.477	**0.122**	ZA19-509	146.505	146.381	**0.124**
41233	150.345	150.223	**0.122**	ZA21-507	150.215	150.091	**0.124**
41236	158.006	157.88	**0.122**	ZA23-505	152.987	152.86	**0.122**

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41253	154.565	154.444	**0.121**	ZA23-511	151.228	151.106	**0.122**
41256	152.883	152.762	**0.121**	ZAZB-520	150.363	150.244	**0.119**
41268	161.607	161.486	**0.121**	ZBZC-509	152.171	152.050	**0.121**
41269	157.207	157.086	**0.121**	ZBZC-539	155.144	155.023	**0.121**
41270	153.198	153.077	**0.121**	ZBZC-540	154.788	154.667	**0.121**
41271A	155.323	155.202	**0.121**	ZBZC-541	154.803	154.682	**0.121**
41285A	152.886	152.768	**0.118**	ZBZC-544	154.266	154.145	**0.121**
41285B	153.104	152.985	**0.119**	ZC7-513	161.772	161.650	**0.122**
41285C	150.659	150.540	**0.119**	ZNSBA2-507	134.568	134.443	**0.125**
41285D	150.786	150.667	**0.119**	ZNSBA11-502	135.559	135.438	**0.121**
41285E	151.402	151.283	**0.119**	4424BA-1021	128.801	128.688	**0.113**
41299	168.104	167.984	**0.120**	4424BA-1024	134.823	134.707	**0.116**
41300	169.601	169.481	**0.120**	C8-516	212.311	212.189	**0.122**
41301	169.972	169.852	**0.120**	CZA-610	167.299	167.175	**0.124**
41302	168.127	168.007	**0.120**	CZC-531	167.288	167.166	**0.122**

Another graphical representation of the MSL vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ by using orthographic projection is illustrated in the Fig. 12.

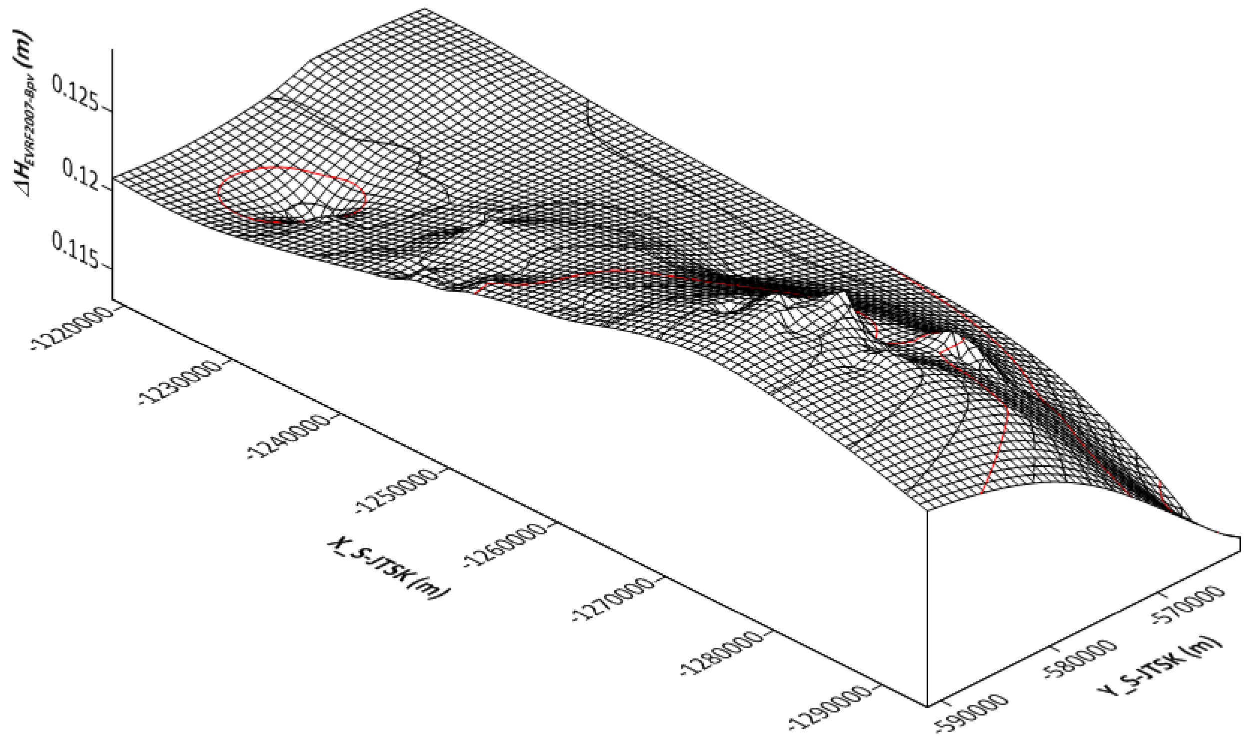

Fig. 12. Vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ (m) between European vertical reference frame (EVRF2007, Amsterdam) and Slovakian Baltic vertical reference frame (Bpv, Kronštadt) on the border river Morava in orthographic projection.

The mean value of the MSL vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ on all Austrian and Slovakian levelling points (together 114 points) is **0.124 m**. The positive values of these MSL vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ mean that the initial point in the European vertical reference frame EVRF2007 (point with normal MSL heights $H_{\mathrm{EVRF2007}} = 0$ m in Amsterdam) is lower with respect to the initial point in Slovakian Baltic vertical reference frame Bpv (point with normal height $H_{Bpv} = 0$ m in Kronstadt). In the Tab. 10 are demonstrated the residuals on individual levelling points. The root mean square (RMS) error **σ_0 = 3 mm** and mean error of arithmetic average **σ = 0.3 mm** of vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ was computed from these residuals.

Tab. 10 Residuals of MSL vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ on Austrian and Slovakian levelling points.

Austrian levelling points – 57 points			Slovakian levelling points – 57 points		
Point ID	**$\Delta H_{EVRF2007\text{-}Bpv}$ (m)**	**Residuals (m)**	**Point ID**	**$\Delta H_{EVRF2007\text{-}Bpv}$ (m)**	**Residuals (m)**
38840 C	0.128	-0.004	3441MA -53	0.120	0.004
38840 D	0.128	-0.004	3443MA -42	0.125	-0.001
38840 E	0.128	-0.004	4422BA-1002	0.129	-0.005
38849	0.127	-0.003	AZR-529	0.121	0.003
38856	0.128	-0.004	C59-509	0.121	0.003
38867	0.127	-0.003	CZA-501	0.125	-0.001
38868 D	0.126	-0.002	CZA-503	0.124	0.000
41074	0.128	-0.004	CZA-505	0.124	0.000
41075	0.128	-0.004	CZA-506	0.125	-0.001
41075 B	0.127	-0.003	CZA-508	0.125	-0.001
41076	0.129	-0.005	CZA-509	0.126	-0.002
41095 C	0.128	-0.004	CZA-520	0.125	-0.001
41096	0.128	-0.004	CZA-544	0.125	-0.001
41096 A	0.127	-0.003	CZA-644	0.122	0.002
41097	0.127	-0.003	CZA-662.01	0.121	0.003
41097 A	0.127	-0.003	CZA-673	0.121	0.003
41098	0.127	-0.003	ZA1-505	0.126	-0.002
41099	0.127	-0.003	ZA1-510	0.126	-0.002
41102	0.128	-0.004	ZA1-516	0.127	-0.003
41103	0.127	-0.003	ZA1-521	0.128	-0.004
41138 A	0.126	-0.002	ZA1-522	0.128	-0.004
41138 B	0.126	-0.002	ZA1-525	0.128	-0.004
41138 C	0.126	-0.002	ZA1-526	0.128	-0.004

41138 D	0.126	-0.002	ZA1-537	0.129	-0.005
41184 A	0.125	-0.001	ZA1-539	0.129	-0.005
41184 B	0.125	-0.001	ZA2-500	0.128	-0.004
41184 C	0.125	-0.001	ZA2-501	0.128	-0.004
41184 D	0.125	-0.001	ZA2-502	0.129	-0.005
41184 E	0.125	-0.001	ZA2-503	0.129	-0.005
41201	0.124	0.000	ZA5-505	0.128	-0.004
41202	0.124	0.000	ZA10-517	0.126	-0.002
41203	0.124	0.000	ZA10-518	0.126	-0.002
41209 A	0.124	0.000	ZA10-520	0.126	-0.002
41210 A	0.123	0.001	ZA18-511.02	0.123	0.001
41211 A	0.123	0.001	ZA18-513	0.123	0.001
41218	0.123	0.001	ZA18-513.02	0.123	0.001
41219	0.123	0.001	ZA18-530	0.124	0.000
41220 A	0.123	0.001	ZA18-532	0.124	0.000
41221 A	0.123	0.001	ZA18-537	0.125	-0.001
41232 A	0.122	0.002	ZA19-509	0.124	0.000
41233	0.122	0.002	ZA21-507	0.124	0.000
41236	0.122	0.002	ZA23-505	0.122	0.002
41253	0.121	0.003	ZA23-511	0.122	0.002
41256	0.121	0.003	ZAZB-520	0.119	0.005
41268	0.121	0.003	ZBZC-509	0.121	0.003
41269	0.121	0.003	ZBZC-	0.121	0.003

			539		
41270	0.121	0.003	ZBZC-540	0.121	0.003
41271 A	0.121	0.003	ZBZC-541	0.121	0.003
41285 A	0.118	0.006	ZBZC-544	0.121	0.003
41285 B	0.119	0.005	ZC7-513	0.122	0.002
41285 C	0.119	0.005	ZNSBA2-507	0.125	-0.001
41285 D	0.119	0.005	ZNSBA11-502	0.121	0.003
41285 E	0.119	0.005	4424BA-1021	0.113	0.011
41299	0.120	0.004	4424BA-1024	0.116	0.008
41300	0.120	0.004	C8-516	0.122	0.002
41301	0.120	0.004	CZA-610	0.124	0.000
41302	0.120	0.004	CZC-531	0.122	0.002

Exact numerical values of the MSL height differences $\Delta H_{EVRF2007\text{-}Bpv}$ for all 315 points located in the middle of border river Morava are not illustrated here due to spatial reasons. Their values range in the interval of **0.113 m** – **0.130 m** (their spread is **0.027 m**).

6. Computation of the MSL height differences $\Delta H_{EVRF2007\text{-}Adria}$ between European vertical reference frame EVRF2007 and currently valid Austrian Adriatic vertical reference frame Adria (System der Normalorthometrischen Höhen mit Pegel Adria Triest 1875) in the middle of border river Morava

Although the MSL vertical differences $\Delta H_{EVRF2007\text{-}Adria} = H_{EVRF2007} - H_{Adria}$ (further in text also referred to as $\Delta H_{EVRF2007\text{-}Adria}$) between European vertical reference frame EVRF2007 and currently valid Austrian Adriatic vertical reference frame Adria are not so important with respect to the use of the vertical reference frames in Slovakia (the mandatory vertical reference frame in Slovakia is Bpv only), their determination in the border area between Austria and Slovakia is important with respect to the exchange of geodata within different interstate and international projects (like mapping of the river´s bed of the river Danube at the border, since the Danube river is a river of European meaning).

At a glance we could estimate the vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ in the area of interest (border river Morava between Austria and Slovakia) by taking into account the average values of vertical differences between Austrian Adriatic vertical reference frame Adria and Slovakian Baltic vertical reference frame Bpv $\Delta H_{Adria\text{-}Bpv}$ (0.534 m, see page 29), in combination with the average value of vertical differences between European vertical reference frame EVRF2007 and Slovakian Baltic vertical reference frame Bpv $\Delta H_{EVRF2007\text{-}Bpv}$ (0.124 m, see page 35). This would result in an average vertical difference $\Delta H_{EVRF2007\text{-}Adria}$ between European vertical reference frame EVRF2007 and Austrian Adriatic vertical reference frame Adria on the level of -0.410 m. The negative sign means that initial point of Austrian vertical reference frame Adria in Trieste, Italy is higher than the initial point of European vertical reference frame EVRF2007 located in Amsterdam, The Netherlands.

However, we decided to verify such estimation by real computation of the vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ on the 315 individual points located in the middle of the border Morava river by the same manner like described in Chapters 4 and 5 using the same input data set (57 Austrian levelling points and 52 Slovakian levelling points, since 5 earlier mentioned Slovakian levelling points lie on the edge of the area covered by Austrian geoid model and cannot be used for a reliably interpolation).

In short, we utilized the Adriatic normal orthometric MSL heights determined by precise levelling method on the Austrian levelling points. We also determined their normal EVRF2007 MSL heights by using Slovakian quasi-geoid model called DMQSK2014-E, ver. 06.2014 which has sufficient overlapping into Austria at this border area. This quasi-geoid model was also used to determine the normal EVRF MSL heights on Slovakian levelling points and their Adriatic normal orthometric MSL heights were determined by using Austrian geoid model. In the Tab. 11, the vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ on all 109 Austrian and Slovakian levelling points are demonstrated. By this way, we got the input data for the determination of vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ on the individual 315 points located in the middle of Morava river (they are each other 250 m away) by applying Kriging´s interpolation method.

Tab. 11 Vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ on Austrian and Slovakian levelling points.

Austrian levelling points – 57 point				**Slovakian levelling points – 52 points**			
Point ID	$\boldsymbol{H_{EVRF2007}}$ **(m)**	**HAdria (m)**	$\boldsymbol{\Delta H_{EVRF2007\text{-}Adria}}$ **(m)**	**Point ID**	**$H_{EVRF2007}$ (m)**	**H_{Adria} (m)**	$\boldsymbol{\Delta H_{EVRF2007\text{-}Adria}}$ **(m)**
38840C	197.031	197.460	**-0.429**	3441MA-53	154.687	155.115	**-0.428**
38840 D	223.888	224.317	**-0.429**	3443MA-42	148.819	149.246	**-0.427**
38840E	285.366	285.792	**-0.426**	4422BA-1002	256.850	257.278	**-0.428**
38849	161.186	161.604	**-0.418**	AZR-529	134.463	134.699	**-0.236**
38856	148.106	148.519	**-0.413**	C59-509	173.380	173.812	**-0.432**
38867	140.838	141.247	**-0.409**	CZA-501	141.648	141.983	**-0.335**
38868 D	141.280	141.688	**-0.408**	CZA-503	152.885	153.235	**-0.350**
41074	144.271	144.70	**-0.433**	CZA-505	159.801	160.15	**-0.358**

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41075	141.004	141.437	**-0.433**	CZA-506	162.938	163.297	**-0.359**
41075B	144.169	144.603	**-0.434**	CZA-508	166.408	166.765	**-0.357**
41076	143.296	143.730	**-0.434**	CZA-509	176.178	176.537	**-0.359**
41095C	147.968	148.390	**-0.422**	CZA-520	188.754	189.146	**-0.392**
41096	149.938	150.358	**-0.420**	CZA-544	217.933	218.347	**-0.414**
41096A	149.712	150.133	**-0.421**	CZA-644	156.004	156.399	**-0.395**
41097	145.499	145.920	**-0.421**	CZA-662.01	151.467	151.858	**-0.391**
41097A	143.832	144.253	**-0.421**	CZA-673	152.318	152.707	**-0.389**
41098	140.812	141.232	**-0.420**	ZA1-505	140.848	141.260	**-0.412**
41099	139.805	140.224	**-0.419**	ZA1-510	143.264	143.678	**-0.414**
41102	150.211	150.630	**-0.419**	ZA1-516	152.649	153.066	**-0.417**
41103	150.202	150.622	**-0.420**	ZA1-521	147.216	147.638	**-0.422**
41138A	144.266	144.688	**-0.422**	ZA1-522	157.940	158.362	**-0.422**
41138B	142.873	143.294	**-0.421**	ZA1-525	152.847	153.270	**-0.423**
41138C	143.249	143.672	**-0.423**	ZA1-526	149.569	149.994	**-0.425**
41138D	141.448	141.870	**-0.422**	ZA1-537	142.377	142.801	**-0.424**
41184A	144.399	144.824	**-0.425**	ZA1-539	145.691	146.114	**-0.423**
41184B	143.647	144.072	**-0.425**	ZA2-500	149.846	150.268	**-0.422**
41184C	143.907	144.333	**-0.426**	ZA2-501	151.105	151.525	**-0.420**
41184D	144.492	144.918	**-0.426**	ZA2-502	152.042	152.459	**-0.417**
41184E	140.685	141.111	**-0.426**	ZA2-503	143.576	143.991	**-0.415**
41201	149.434	149.85	**-0.422**	ZA5-505	164.122	164.55	**-0.428**

		6				0	
41202	148.710	149.130	**-0.420**	ZA10-517	145.461	145.877	**-0.416**
41203	150.328	150.748	**-0.420**	ZA10-518	144.748	145.164	**-0.416**
41209 A	173.377	173.805	**-0.428**	ZA10-520	145.210	145.626	**-0.416**
41210 A	175.568	175.994	**-0.426**	ZA18-511.02	145.568	145.981	**-0.413**
41211 A	167.200	167.626	**-0.426**	ZA18-513	145.851	146.262	**-0.411**
41218	150.019	150.437	**-0.418**	ZA18-513.02	145.745	146.158	**-0.413**
41219	148.815	149.229	**-0.414**	ZA18-530	146.536	146.955	**-0.419**
41220 A	148.977	149.393	**-0.416**	ZA18-532	145.440	145.860	**-0.420**
41221 A	145.713	146.128	**-0.415**	ZA18-537	144.091	144.513	**-0.422**
41232 A	152.599	153.014	**-0.415**	ZA19-509	146.505	146.893	**-0.388**
41233	150.345	150.755	**-0.410**	ZA21-507	150.215	150.594	**-0.379**
41236	158.006	158.419	**-0.413**	ZA23-505	152.987	153.357	**-0.370**
41253	154.565	154.971	**-0.406**	ZA23-511	151.228	151.626	**-0.398**
41256	152.883	153.288	**-0.405**	ZAZB-520	150.363	150.790	**-0.427**
41268	161.607	162.033	**-0.426**	ZBZC-509	152.171	152.572	**-0.401**
41269	157.207	157.634	**-0.427**	ZBZC-539	155.144	155.512	**-0.368**
41270	153.198	153.622	**-0.424**	ZBZC-540	154.788	155.153	**-0.365**
41271 A	155.323	155.754	**-0.431**	ZBZC-541	154.803	155.168	**-0.365**
41285 A	152.886	153.318	**-0.432**	ZBZC-544	154.266	154.630	**-0.364**
41285B	153.104	153.532	**-0.428**	ZC7-513	161.772	162.124	**-0.352**
41285C	150.659	151.086	**-0.427**	ZNSBA2-507	134.568	134.984	**-0.416**
41285	150.786	151.21	**-0.427**	ZNSBA11	135.559	135.79	**-0.237**

D		3		-502		6	
41285E	151.402	151.829	**-0.427**	4424BA-1021*	Not used	Not used	Not used
41299	168.104	168.543	**-0.439**	4424BA-1024*	Not used	Not used	Not used
41300	169.601	170.040	**-0.439**	C8-516*	Not used	Not used	Not used
41301	169.972	170.412	**-0.440**	CZA-610*	Not used	Not used	Not used
41302	168.127	168.568	**-0.441**	CZC-531*	Not used	Not used	Not used

The average value of vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ on all Austrian and Slovakian points (together 109 points) is **-0.410 m**. The negative values of vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ mean that the initial point in the European vertical reference frame EVRF2007 (point with normal MSL heights $H_{\mathrm{EVRF2007}} = 0$ m in Amsterdam, The Netherlands) is higher with respect to the initial point in Austrian Adriatic vertical reference frame Adria (point with normal orthometric height $H_{Adria} = 0$ m in Trieste, Italy).

The residuals on all levelling points are presented in the Tab. 12. From these values were computed the root mean square error (RMS) **σ_0 = 32 mm** and mean error of arithmetic average **σ = 2 mm** of vertical differences $\Delta H_{EVRF2007\text{-}Adria}$.

Tab. 12 Residuals of vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ on Austrian and Slovakian levelling points.

Austrain levelling points – 57 points			**Slovakian levelling points – 52 points**		
Point ID	**$\Delta H_{EVRF2007\text{-}Adria}$ (m)**	**Residuals (m)**	**Point ID**	**$\Delta H_{EVRF2007\text{-}Adria}$ (m)**	**Residuals (m)**
38840C	**-0.429**	0.019	3441MA-53	**-0.428**	0.018
38840D	**-0.429**	0.019	3443MA-42	**-0.427**	0.017
38840E	**-0.426**	0.016	4422BA-1002	**-0.428**	0.018
38849	**-0.418**	0.008	AZR-529	**-0.236**	-0.174
38856	**-0.413**	0.003	C59-509	**-0.432**	0.022
38867	**-0.409**	-0.001	CZA-501	**-0.335**	-0.075
38868D	**-0.408**	-0.002	CZA-503	**-0.350**	-0.060
41074	**-0.433**	0.023	CZA-505	**-0.358**	-0.052
41075	**-0.433**	0.023	CZA-506	**-0.359**	-0.051
41075B	**-0.434**	0.024	CZA-508	**-0.357**	-0.053
41076	**-0.434**	0.024	CZA-509	**-0.359**	-0.051

41095 C	**-0.422**	0.012	CZA-520	**-0.392**	-0.018
41096	**-0.420**	0.010	CZA-544	**-0.414**	0.004
41096 A	**-0.421**	0.011	CZA-644	**-0.395**	-0.015
41097	**-0.421**	0.011	CZA-662.01	**-0.391**	-0.019
41097 A	**-0.421**	0.011	CZA-673	**-0.389**	-0.021
41098	**-0.420**	0.010	ZA1-505	**-0.412**	0.002
41099	**-0.419**	0.009	ZA1-510	**-0.414**	0.004
41102	**-0.419**	0.009	ZA1-516	**-0.417**	0.007
41103	**-0.420**	0.010	ZA1-521	**-0.422**	0.012
41138 A	**-0.422**	0.012	ZA1-522	**-0.422**	0.012
41138 B	**-0.421**	0.011	ZA1-525	**-0.423**	0.013
41138 C	**-0.423**	0.013	ZA1-526	**-0.425**	0.015
41138 D	**-0.422**	0.012	ZA1-537	**-0.424**	0.014
41184 A	**-0.425**	0.015	ZA1-539	**-0.423**	0.013
41184 B	**-0.425**	0.015	ZA2-500	**-0.422**	0.012
41184 C	**-0.426**	0.016	ZA2-501	**-0.420**	0.010
41184 D	**-0.426**	0.016	ZA2-502	**-0.417**	0.007
41184 E	**-0.426**	0.016	ZA2-503	**-0.415**	0.005
41201	**-0.422**	0.012	ZA5-505	**-0.428**	0.018
41202	**-0.420**	0.010	ZA10-517	**-0.416**	0.006
41203	**-0.420**	0.010	ZA10-518	**-0.416**	0.006
41209 A	**-0.428**	0.018	ZA10-520	**-0.416**	0.006
41210 A	**-0.426**	0.016	ZA18-511.02	**-0.413**	0.003
41211 A	**-0.426**	0.016	ZA18-513	**-0.411**	0.001
41218	**-0.418**	0.008	ZA18-513.02	**-0.413**	0.003

41219	**-0.414**	0.004	ZA18-530	**-0.419**	0.009
41220A	**-0.416**	0.006	ZA18-532	**-0.420**	0.010
41221A	**-0.415**	0.005	ZA18-537	**-0.422**	0.012
41232A	**-0.415**	0.005	ZA19-509	**-0.388**	-0.022
41233	**-0.410**	0.000	ZA21-507	**-0.379**	-0.031
41236	**-0.413**	0.003	ZA23-505	**-0.370**	-0.040
41253	**-0.406**	-0.004	ZA23-511	**-0.398**	-0.012
41256	**-0.405**	-0.005	ZAZB-520	**-0.427**	0.017
41268	**-0.426**	0.016	ZBZC-509	**-0.401**	-0.009
41269	**-0.427**	0.017	ZBZC-539	**-0.368**	-0.042
41270	**-0.424**	0.014	ZBZC-540	**-0.365**	-0.045
41271A	**-0.431**	0.021	ZBZC-541	**-0.365**	-0.045
41285A	**-0.432**	0.022	ZBZC-544	**-0.364**	-0.046
41285B	**-0.428**	0.018	ZC7-513	**-0.352**	-0.058
41285C	**-0.427**	0.017	ZNSBA2-507	**-0.416**	0.006
41285D	**-0.427**	0.017	ZNSBA11-502	**-0.237**	-0.173
41285E	**-0.427**	0.017	4424BA-1021*	Not used	Not used
41299	**-0.439**	0.029	4424BA-1024*	Not used	Not used
41300	**-0.439**	0.029	C8-516*	Not used	Not used
41301	**-0.440**	0.030	CZA-610*	Not used	Not used
41302	**-0.441**	0.031	CZC-531*	Not used	Not used

Graphical shape of vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ for all individual 315 points located in the middle of Morava river is illustrated in the Fig. 13.

Exact numerical values of vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ for all 315 points are not illustrated here due to spatial reasons. Their values range in the interval of **0.391 m** to **0.429 m** (their spread is **0.038 m**).

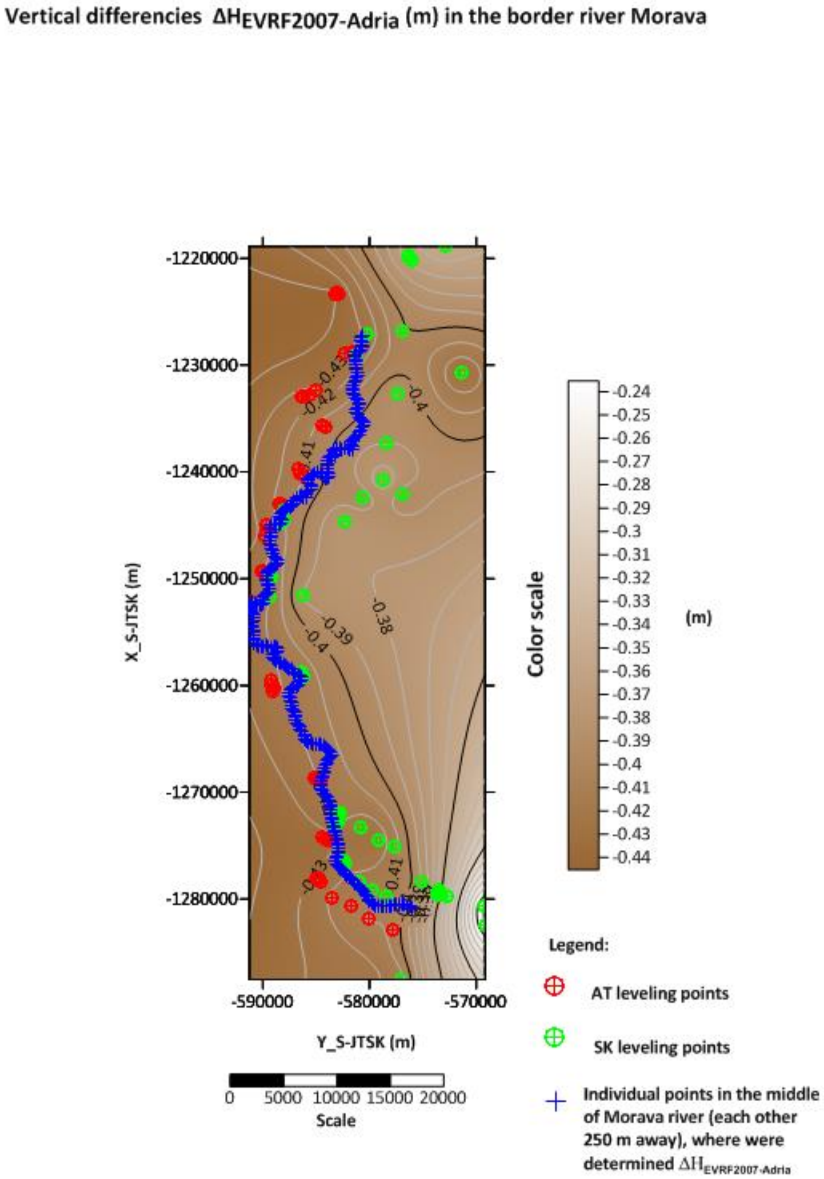

Fig. 13. Vertical differences $\Delta H_{EVRF2007\text{-}Adria)}$ (m) between European vertical reference frame (EVRF2007, Amsterdam) and Austrian Adriatic reference frame (Adria, Trieste) on the border river Morava.

Another graphical representation of the MSL vertical differences $\Delta H_{EVRF2007\text{-}Bpv}$ by using orthographic projection is demonstrated in the Fig. 14.

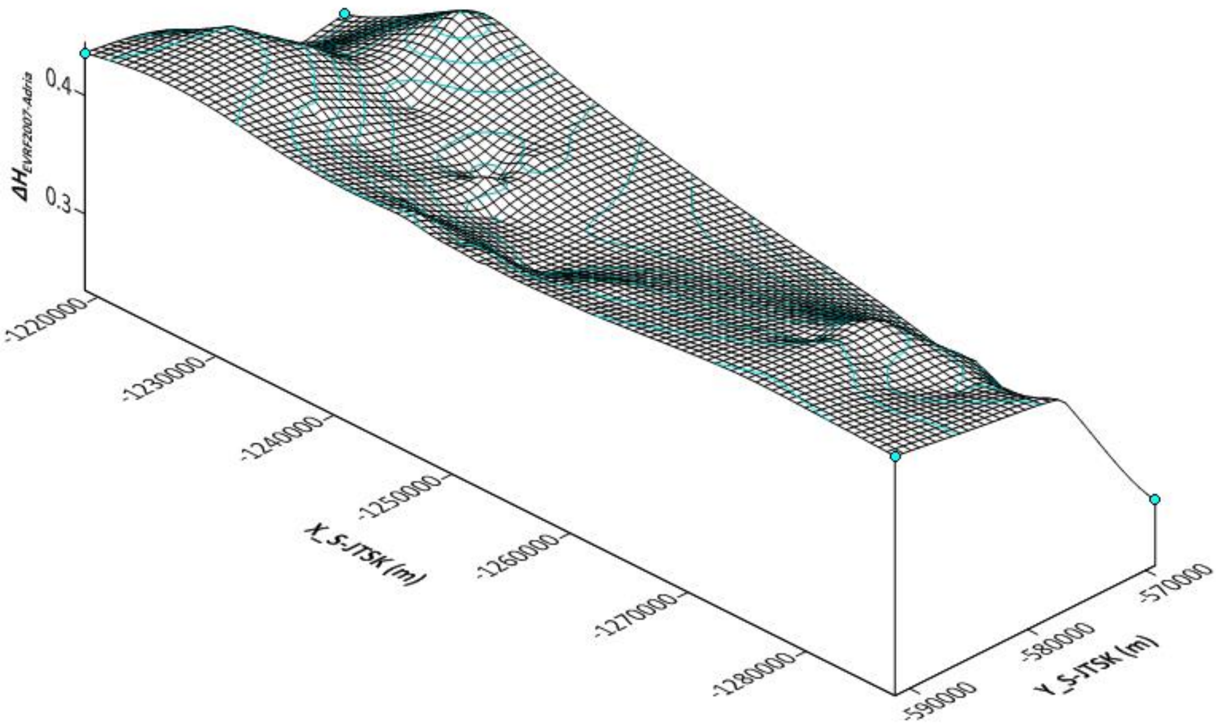

Fig. 14. Vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ (m) European vertical reference frame (EVRF2007, Amsterdam) and Austrian Adriatic vertical reference frame (Adria, Trieste) on the border river Morava in orthographic projection.

7. CONCLUSION

The determination of optimal MSL vertical differences between two currently valid vertical reference frames requires to have available a sufficient data set of levelling points with their currently valid respective MSL values. In the case, when such set of levelling points is not available, it is necessary to realize observations for their determination in the area of interest. In addition, fur such kind of computations it is necessary to have available as precise relevant (quasi)geoid models as possible, which would cover the whole area of interest.

Statistical evaluation of achieved results presented in this article is summarized in the Tab. 13. The assessing parameters reflect uncertainties in the precise levelling method and in the heterogeneity of used levelling points (for example, Slovakian set of levelling points is a mix of levelling points of 1st and 2nd order), as well as uncertainties in the determination of Austrian geoid and Slovakian quasi-geoid.

Tab. 13 Statistical evaluation of achieved results in the area of interest (border river Morava with total length of about 78.5 km).

Statistical parameters of assessment of achieved results	**Vertical MSL differences ΔH (m) between individual vertical reference frames Bpv, Adria and EVRF2007 in the area of border river Morava**		
	$\Delta H_{Adria\text{-}Bpv}$	$\Delta H_{EVRF2007\text{-}Bpv}$	$\Delta H_{EVRF2007\text{-}Adria}$
Average value	**0.534**	**0.124**	**- 0.410**
Root mean square (RMS) error σ_0	**0.033**	**0.003**	**0.032**
Mean error of arithmetic average σ	**0.003**	**0.0003**	**0.002**

The estimated accuracy of the computation of the vertical MSL differences $\Delta H_{Adria\text{-}Bpv}$ between Austrian Adriatic vertical reference frame (System der Normalorthometrischen Höhen mit Pegel Adria Triest 1875) and Slovakian Baltic vertical reference frame Bpv (Kronstadt) by using both respective models of Austrian geoid and Slovakian quasi-geoid is on the level of technical levelling. The same can be said for the determination of vertical MSL differences $\Delta H_{EVRF2007\text{-}Bpv}$ between European vertical reference frame EVRF2007 (Amsterdam) and Slovakian Baltic vertical reference frame Bpv (Kronstadt). Their mean value in the area of interest (border river Morava) is 0.124 m (see Tab. 8). The value is in very good accord with the estimated value of 0.120 m for the territory of Slovakia declared by (*Sacher, Ihde, Seeger, 1998*).

The mean value of the determined MSL vertical differences $\Delta H_{EVRF2007\text{-}Adria}$ between European vertical reference frame EVRF2007 (Amsterdam) and Austrian Adriatic reference frame Adria is –0.410 m (see Tab. 12). It differs from the value of –0.350 cm declared in (*Sacher, Ihde, Seeger, 1998*) for the whole Austrian territory by 6 cm.

Another alternative would be the realisation of precise levelling observations on identical levelling points on the Austrian side of Morava and Danube river with the initial point on Slovakian levelling point of the 1st order with currently valid MSL height in the Slovakian Bpv vertical reference frame. This would result in the determination of MSL normal heights on the selected set of Austrian levelling points and vice versa. It means, the initial point for such precise levelling observations would be a point on the Austrian side with currently valid Adriatic MSL height. By this, there would be determined precise Adriatic MSL normal orthometric heights on the selected set of Slovakian levelling points. The same procedure would also be applied for the determination of MSL heights in the European vertical reference frame EVRF2007.

This would assign that all MSL vertical differences on the chosen set of levelling points on both sides would be computed from directly observed data (not from geoid and quasi-geoid models) and this data would serve as an input values for the determination of all MSL vertical differences in the area of interest by using the Kriging´s interpolation method for any point of interest in all three vertical reference frames (Bpv, EVRF2007 and Adria). Using this way of the determination of the vertical MSL differences on the chosen set of levelling points would guarantee the highest possible level of accuracy on the millimetre level. However, its main and big disadvantage is its time and financial demand.

Despite it, it is necessary to consider that the availability of high quality vertical MSL differences are imperative pre-requisites for reliable determination of impact of the variation of water masses on the surrounding area and its infrastructure. Therefore, it would be suitable and highly beneficial to verify the achieved results by direct observations in the field. Their additional and very important advantage would be essential simplification and facilitation of geodata interchange and elimination (or at least great reduction) of possible sources of errors, which mostly emerge because of the interchange of very heterogeneously data within different interstate and international projects.

Any improvement of precise levelling and gravimetric observations bring refinement in the complex realisation of a vertical reference frame (see also Liebsch, Rülke, Sacher, Ihde, 2014) and leads to the improvement of the determination of the MSL vertical differences between different vertical reference frames. This also heads to the more precise assessment of earth´s surface dynamics.

In 2015, a group of authors (*Ihde, et al, 2015*) stated that during next four years will be established a commission within IAG, which will examine the possibility of replacement of currently valid Global Reference System 1980 (GRS80) by a new Geodetical Reference System (GRS). This working group prepared a proposal of total new parameters for the new GRS and this proposal was presented on the IUGG plenary session in Montreal, Canada in the time of July 8 – July 18, 2019.

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LITERATURE

BENEŠ, F.: Současný stav výškových geodetických základů. Geodetický a kartografický obzor, ročník 45/87, 1999, číslo 6, str. 113 – 117. (in czech)

BÖHM, J., HORA, L., KOLENATÝ, E.: Vyšší geodézie, díl 2, Ediční středisko ČVUT, Praha 1, 1981. (in czech)

DREWES, H.: Reference Systems, Reference Frames, and Geodetic Datum – Basic Considerations. In: Observing our Changing Earth. IAG Symposia 133, Springer – Verlag Berlin, Heidelberg 2009.

IHDE, J., AUGATH, W.: The Vertical Reference System for Europe. In: Veröff. Bayer Komm. Int.Erdmess. Bayer Akad. d. Wiss., Astron-geod. Arb., München 2000, H. 61, p. 99 – 110.

IHDE, J., MÄKINEN, J., SACHER, M.: Conventions for the Definition and Realization of a European Vertical Reference System (EVRS). IAG SC 1.3 a EUREF, 2008.

JEKELI Ch.: Geometric Reference Systems in Geodesy. Division of geodesy and Geospatial Science, School of Earth Sciences, Ohio State University, July 2006.

KRUIS, B.: Srovnávací studium nivelačních horizontů ČSR a okolních států. Geodetický a kartografický sborník, 1957. (in czech)

LIEBSCH, G., RÜLKE, A., SACHER, M., IHDE, J.: Definition and Realization of the EVRS: How do we want to proceed? Presented at EUREF Symposium 2014, June 04-06, Vilnius, Lithuania.

LOWRIE, W.: Fundamentals of Geophysics, 2nd edition., Cambridge: Cambridge University Press, 381 pp., 2007)

MILIČKA, J.: Uhľovodíkový potenciál Podunajskej panvy. Univerzita Komenského, Prírodovedecká fakulta, 2017. (in slovak)

MOJZEŠ, M., KALAFUT, M.: Technická správa – Diferencie medzi Jadranským vertikálnym referenčným systémom realizovaným ČSJNS (Československá jednotná nivelačná sieť) – Jadran$_{ČSJNS}$ a Baltským vertikálnym referenčným systémom realizovaným vertikálnou referenčnou sieťou Bpv (Balt po vyrovnaní). Bratislava, Máj 2018. (in slovak)

PICK, M.: Výšky, nivelace, hladinové a vztažné plochy. Zborník referátov „Súčasný stav a perspektívy rozvoja geodézie a využívanie výsledkov vedy a výskumu v geodetickej praxi". Bardejovské Kúpele, 3.9.1987. (in czech)

SACHER, M., LIEBSCH, G.: Short description of the European vertical reference system and its realizations. EUREF Symposium 2015, June 03-05, Leipzig, Germany.

SACHER, M., IHDE, J., SEEGER, H.: Preliminary Transformation Relations between National European Height Systems and UELN. Oslo, CERCO – Plenary 1998. 19s.

SAV – mapa úplných Bouguerových gravitačných anomálií. (in slovak)

VYKUTIL, J.: Vyšší geodézie, Díl II. Ediční středisko VUT Brno, 1978. (in czech)

VYKUTIL, J.: Výpočet tíhových korekcí nivelace v baltském výškovém systému. Geodetický a kartografický obzor. Sv. 5/47 (1959) č. 8. (in czech)