Triangulation of Delaunay: Application to the deformation monitoring of geodetic network by use of strain tensors

Attaouia BELHADJ and Boualem GHEZALI and Salem KAHLOUCHE, Algeria

SUMMARY

This paper aims to present a deformation analysis methodology, of a geodetic network monitoring of industrial infrastructures or active zones, based on the computation of strain tensors and their significance degrees, under infinitesimal deformation of a continuous, homogeneous and elastic medium hypothesis. The geodetic observation techniques (terrestrial or spatial) dedicated to monitoring, lead to the assessment of absolute or relative; temporal displacements, of points set constructed on the site. The objective consists therefore to pass from displacement information to deformation information with its confidence degree or significance. The study of the strain tensor is classical, but its interpretation is not direct. Usually the decomposition of the strain deformation in symmetrical and anti-symmetrical tensor, furnishes the deformation rate parameters, such as: dilation, shear and differential rotation. The methodology adopted has two main steps: the first one is the representation of the strain tensors and their primitives according to Delaunay Triangulation, and the second concerns the assessment of the confidence degrees of the deformation primitives estimated by Monte Carlo method.The application focused on the auscultation network of the LNG underground reservoir GL4Z industrial complex of Arzew (Algeria). The data used are obtained from two GPS campaigns which were carried out over six years period (February 2000, February 2006). The GPS network consists of 56 points on the ground surrounding the reservoir. The data concern the local horizontal geodetic coordinates (N, E) and the vertical coordinate (U) of the auscultation points, for 2D and 1D network deformation analysis, respectively. The results show that the majority of deformations measured are lower than those which can be detected by network. However, few zones show significant deformations, where amplitudes of the primitives are important, mainly in North-East (Sea side) and South-West (Earth side) of the LNG reservoir. The application of Delaunay triangulation in the specific studies is an interesting tool for deformation monitoring, on one hand, for uniqueness of its representation, and on the other hand, for the advantage that deformation is computed directly from the geodetic points of triangles. A dense and well distributed configuration of geodetic points permits to better apprehend the local deformation with Delaunay triangulation.

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1. INTRODUCTION

The geodetic observation techniques (terrestrial or spatial) dedicated to monitoring, lead to the assessment of absolute or relative; temporal displacements, of a points set constructed on the site. However, the realization of a geodetic network requires the use of points set arbitrarily fixed. In an active site, these choices cannot be verified a priori of reference and have implications on deformations calculation. On the other hand, a displacement field is not sufficient to directly quantify the deformations undergone by the network. To overcome these constraints, a tool can be used: the strain tensor. By calculating the gradient of the displacement field, we free ourselves of the absolute aspect of any referencement geographic since it we working on relative changes of network. The strain tensor, defined by the gradient of the displacement field provides information on the local behavior of a vector displacement field. In practice, the study of deformation in a geodetic network is described by two representations: vector-displacements or strain tensor. The method used in this article is based on the representation of deformation tensors and degree of significance of finites elements triangular (Delaunay triangulation), in hypothesis of small deformations of a continuous medium, homogeneous and elastic. It requires the following stages:

2. CONFIGURATION MODE / DELAUNAY TRIANGULATION

The notion of Delaunay triangulation was proposed in 1934 by the Russian mathematician Boris Delaunay, student of Georgy Voronoi. This is a type partitioning of the points set (Pn), positioned in a plane composed of triangles whose vertices are objects, constituting thus a partition of the convex hull of these objects. In graph theory, this triangulation is defined as the dual graph of Voronoi diagram, where two objects are connected by an edge in the Delaunay triangulation if they belong to adjacent Voronoi regions [Damon C], Figure (1).
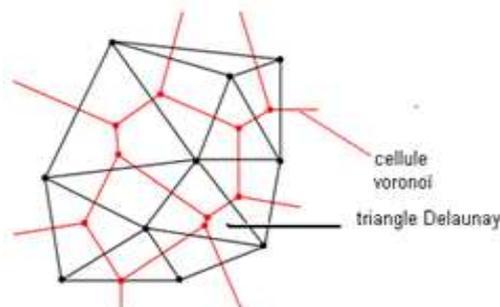

Fig 1: Delaunay Triangulation

The properties of Delaunay triangulation are, Figure (2):
- Let Pi, Pj and Pk three points of a plane (P). The triangle formed by these points is a Delaunay triangle if and only if the circumcircle of this triangle contains no other point of (P).
-Let Pi and Pj be two points of the plan. Then the segment PiPj is an edge of the Delaunay triangulation if and only if there exists a circle passing through Pi and Pj such that the corresponding disk does not contain any other point of (P).
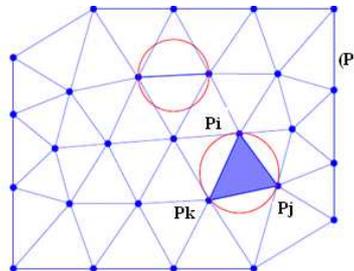

Fig 2: Properties illustration of Delaunay triangulation.

- The Delaunay triangulation of set points in the plane is one that avoids the creation of more flattened triangles: the triangles are, wherever possible, a form close to an equilateral triangle.
- Angular Property: The Delaunay triangulation is among all the triangulations of Euclidean space that maximizes the minimum angle of all triangles [Hétroy F]

- Criteria for the choice of Delaunay triangulation:

In a plane fitted with points in general position (no points aligned or co cyclic), the Delaunay triangulation provides a configuration that does not change. Interest advantageous, since the calculation of local deformations in a geodetic network, changes depending on the elementary figures generated. The Delaunay triangulation solves problem of different results depending on the choice of elementary figures by the uniqueness of his representation [Michel V].

3. METHODOLOGY FOR ANALYSIS DEFORMATIONS

First, it is necessary to clarify the concept of strain tensor and hypotheses in which this study is part. The behavior of a local displacement field is described to help of the strain tensor. By calculating the gradient of the displacement field, one is liberated from the absolute aspect of any geographic referencing. However, the interpretation of a strain tensor is not direct. Its decomposition is necessary for extracting elements or characteristics of deformation parameters. On the other hand, these parameters must be associated with uncertainty coming from errors affecting the displacement vectors. For this purpose it is necessary to define the degree of significance or confidence that we can give to deformation evaluated.
The methodology adopted for the processing and analysis of deformations in a geodetic network based on the use of strain tensor and the degree of significance, assuming deformations are infinitesimal and in a continuous medium, homogeneous and elastic. The calculation of strain tensor is performed on elementary figures triangular (Delaunay triangulation). The network observed in the two periods is assumed to remain exactly the

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same (same points, same measured and same precisions of observations).
The study of deformation in a geodetic network, described in this article involved two main phases of execution:

First Phase: on the Delaunay triangulation calculation for each triangle displacement field, deformation tensor and these primitives with the representation of results obtained

Second Phase: Significance of deformation tensor with evaluation of the uncertainties real (i.e. Measured) of deformation tensor, caused by displacement vectors uncertainties by the Monte Carlo method,study of the network deformability and evaluation of the confidence degree or degree of significance that we can give to deformations calculated.

3.1 **Calculation of deformation tensor on Delaunay triangulation**

In small deformations hypothesis, calculating the deformation tensor on the Delaunay triangulation   requires the following steps:

3.1.1   Computation of displacement field
The vertices displacement vectors (2D and 1D) of each triangle represent   the displacement components points of the geodetic network observed between two epochs.

3.1.2   Computation of deformation tensor

- **Case of horizontal deformation (2D)**: The calculation of the deformation is made for each triangle of Delaunay network. To each triangle corresponds three displacement vectors (points geodetic network) located at the three vertices of the triangle.
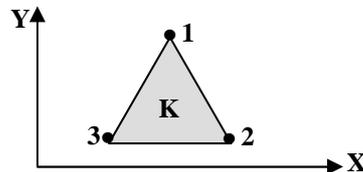Consider the triangle K, Figure (3):

Fig 3: Formalism triangle of network Delaunay.

The function of the local displacement field (u, v) can be represented by a polynomial of first degree in position (X, Y) such that:

$$\begin{cases} u_i(X_i, Y_i) = dX_i = a_1 + a_2 X_i + a_3 Y_i \\ v_i(X_i, Y_i) = dY_i = a_4 + a_5 X_i + a_6 Y_i \end{cases} \quad \text{With } 1 \le i \le 3$$

If we relate the coordinates of the three triangle vertices, with   their displacement vectors to center of gravity triangle [Akrour B] [Million C]. Equations system above became (1):

$$\begin{cases} u_i(x_i, y_i) = dx_i = a_2 x_i + a_3 y_i \\ v_i(x_i, y_i) = dy_i = a_5 x_i + a_6 y_i \end{cases} \quad 1 \le i \le 3 \quad (1)$$

(X0, Y0) and (dX0, dY0) represent respectively the coordinates and displacement related to the gravity center of the triangle K. In Matrix form, equations of motion in (x) and (y) are:

$$N_{(3,2)} P_{(2,1)} = R_{(3,1)} \qquad (2) \qquad N_{(3,2)} G_{(2,1)} = T_{(3,1)} \qquad (3)$$

With:
$$N_{(3,2)} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \\ x_3 & y_2 \end{bmatrix}; \quad P_{(2,1)} = \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}; \quad R_{(3,1)} = \begin{bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{bmatrix}; \quad G_{(2,1)} = \begin{bmatrix} a_5 \\ a_6 \end{bmatrix}; \quad T_{(3,1)} = \begin{bmatrix} dy_1 \\ dy_2 \\ dy_3 \end{bmatrix}$$

The unknown coefficients (a2, a3) and (a5, a6) are adjusted independently by the method of least squares. The deformation tensor (ε) is defined by [Michel V.], and decomposed into a symmetric (S) and antisymmetrical (A) tensor:

$$\varepsilon = \nabla(U) = \frac{\partial U(x,y)}{\partial X} = \begin{bmatrix} \varepsilon_{ux} & \varepsilon_{uy} \\ \varepsilon_{vx} & \varepsilon_{vy} \end{bmatrix} \quad (4) \qquad \text{or} \quad \varepsilon = S + A \quad (5)$$

With
$$S = \begin{bmatrix} \varepsilon_{ux} & \dfrac{\varepsilon_{uy} + \varepsilon_{vx}}{2} \\ \dfrac{\varepsilon_{uy} + \varepsilon_{vx}}{2} & \varepsilon_{vy} \end{bmatrix} \qquad A = \begin{bmatrix} 0 & \dfrac{\varepsilon_{uy} - \varepsilon_{vx}}{2} \\ \dfrac{\varepsilon_{vx} - \varepsilon_{uy}}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \omega \\ -\omega & 0 \end{bmatrix}$$

The components of the deformation tensor for the triangle K are:

$$\begin{cases} \varepsilon_{ux} = \dfrac{\partial u}{\partial x}(x,y) = a_2 \\ \varepsilon_{uy} = \dfrac{\partial u}{\partial y}(x,y) = a_3 \\ \varepsilon_{vx} = \dfrac{\partial v}{\partial x}(x,y) = a_5 \\ \varepsilon_{vy} = \dfrac{\partial v}{\partial y}(x,y) = a_6 \end{cases} \qquad (6)$$

Knowing elements of the deformation tensor (ε_{ux}, ε_{uy}, ε_{vx}, ε_{vy}), we deduce the primitives deformations (dilatation, shear, and differential rotation) defined as follows [Berber M]:

- Dilatation (λ) is the average expansion or contraction at a point

$$\lambda = \frac{1}{2}(\varepsilon_{ux} + \varepsilon_{vy}) \qquad (7)$$

- Total shear (γ) can be defined as the geometric mean of the pure and simple shear:

$$\gamma = \frac{1}{2}\sqrt{(\varepsilon_{ux} - \varepsilon_{vy})^2 + (\varepsilon_{uy} + \varepsilon_{vx})^2} \qquad (8)$$

- Differential rotation ($\delta\omega$): the global rotation of network (Ω) corresponds to average of (Ω) on network. However, the differential rotation $\delta\omega$ is given by the following equation:

$$\delta\omega = \omega - \Omega = \frac{\varepsilon_{uy} - \varepsilon_{vx}}{2} - \Omega \qquad (9)$$

The size of the deformation tensor is expressed by two half axes (a1 and a2) and orientation

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(θ) of first half-axis (a1) , the principal components or principal constraints are λ1 and λ2

$$a_1 = |\lambda_1| \quad et \quad a_2 = |\lambda_2| \quad ; \quad \tan(2\theta) = \frac{\varepsilon_{uy} + \varepsilon_{vx}}{\varepsilon_{vy} - \varepsilon_{ux}} \quad (10)$$

$$\lambda_1 = \frac{\varepsilon_{ux} + \varepsilon_{vy}}{2} + \frac{1}{2}\sqrt{(\varepsilon_{ux} - \varepsilon_{vy})^2 + (\varepsilon_{uy} + \varepsilon_{vx})^2} \; ; \; \lambda_2 = \frac{\varepsilon_{ux} + \varepsilon_{vy}}{2} - \frac{1}{2}\sqrt{(\varepsilon_{ux} - \varepsilon_{vy})^2 + (\varepsilon_{uy} + \varepsilon_{vx})^2}$$

Note that a negative constraint corresponds to a contraction and a positive constraint to expansion.

    – **Case of verticals deformations (1D)**

In the case of small deformations, the function of the local vertical displacement field can be represented by a polynomial of first degree:

$$U(z_i) = dz_i = a_0 + a_1 z_i \qquad (11)$$

dz_i: represents the displacement of the vertical component (z) of point (i) network observed between two epochs.For a polygon (n), the system of equations (11) is written in matrix form (12):

$$\begin{bmatrix} dz_1 \\ . \\ dz_i \\ . \\ dz_n \end{bmatrix} = \begin{bmatrix} 1 & z_1 \\ . & . \\ 1 & z_i \\ . & . \\ 1 & z_n \end{bmatrix} * \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \quad (i = 1, \ldots, n) \quad (12)$$

The deformation tensor ε (unit elongation following (z) is defined by [Berber M] :

$$\varepsilon = \frac{\partial U(z)}{\partial z} = [\varepsilon_{wz}] = a_1 \qquad (13)$$

The adjustment of the coefficients (a0, a1) is carried out by the method of least squares. The coefficient a1 is the elongation unit, evoking swelling (uprising) if the value is positive or settlement (subsidence) if the value is negative.

3.2 Significance of deformation: Monte Carlo method.

The Monte Carlo method determines the significance of deformation tensor. Nevertheless, it seems more consistent to apply it to a random selection of observations according to their statistical rather than displacement vectors. Indeed, it is considered that the observations are uncorrelated, while the same assumption about the displacement vectors is false [Michel V, Person T]. The general idea is to build a set of axes of deformation tensors by disrupting measurements. The ends of these axes are normally distributed around the tensor unperturbed and thus form a point cloud where dispersion of perturbed solutions is an indication of the severity of the unperturbed solution.

The significance of the strain tensor calculated between two epochs corresponds to confidence degree that we can give to calculated values. This level of trust depends on many parameters, namely:

- Uncertainties on the measurements when observing network,
- The process of adjustment by least squares of the network at any given time,

- The process of determining the strain tensor, aspect strictly related to the geometry of the network neighboring the element of the figure considered.

The calculation of significance is based on the deformability (14), calculated by summing the average with tolerance of primitives virtual positive deformation (obtained from a simulation game). Tolerance equal to 2.57 times the standard deviation of primitives can cover 99% of virtual deformations. In the hypothesis that the sample calculated follows a normal law [Michel V, Person T]. Deformability permits to estimate the deformations that can be detected (quantify the influence of uncertainties in terms of deformation).

$$\begin{cases} \lambda_{def} = \overline{\lambda} + 2.57\,\sigma_{\lambda} \\ \gamma_{def} = \overline{\gamma} + 2.57\,\sigma_{\gamma} \\ \delta\omega_{def} = \overline{\delta\omega} + 2.57\,\sigma_{\delta\omega} \end{cases} \qquad (14)$$

Let for each primitive, the significance level (15) is:

$$\begin{cases} \sum_{\lambda} = \dfrac{\lambda - \lambda_{déf}}{\lambda_{déf}} \\ \sum_{\gamma} = \dfrac{\gamma - \gamma_{déf}}{\gamma_{déf}} \\ \sum_{\delta\omega} = \dfrac{|\delta\omega| - \delta\omega_{déf}}{\delta\omega_{déf}} \end{cases} \qquad (15)$$

The degree of significance of the deformation **Σ** takes a value between (-1 +∞), such as:

- **Σ < 0**, the measured strain is less than the deformability of the network, one can assume that the deformation is not significant (DNS) and cannot be considered that a consequence of the observation noise in measurements moments.

- **Σ > 0**, the measured deformation is greater than the deformability of network, one consider that the deformation is significant (DS)

- **0 ≤ Σ < 1**, the deformation is the order of the deformability of the network, while being significant and in the order of the deformability (DS & OD).

4. PRESENTATION OF THE TEST AREA

The tests focused on the determination of deformations in an auscultation network of the reservoir GL4Z - complex LNG - at Arzew. Two campaigns of GPS observations, spaced six (06) years (February 2000, February 2006), were performed. Auscultation network consists of 13 support points away from a hundred meters from the reservoir and 43 secondary benchmarks distributed in the field surrounding the reservoir (Figure 4). The data used in the sets processing of coordinates   are expressed in the local survey reported to the World Geodetic System WGS84.This data are organized by station and presented in an ASCII file (XX.dat). This file contains.

-The local geodetic coordinates of a GPS station (E: horizontal component, N: north horizontal component, U: Vertical component or elevation).

-The standard deviations of the three components (E, N, U).

The horizontal components were used for the deformations analysis in the horizontal plane (2D) and the vertical component was used for the deformation study in the vertical (1D).

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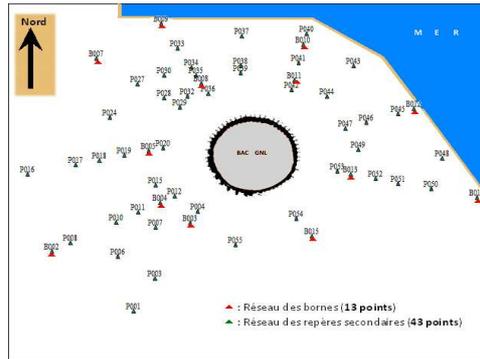Rome, Italy, 6-10 May 2012

Fig.4. Auscultation network configuration

5. Results and analysis

The results analysis of processing data focused on the interpretation of values deformation primitives and degree of significance obtained on finite element triangular "Delaunay triangulation".
 Geodetic data used in the treatment are two types:
– For a 2D network (local behavior in the horizontal plane): it sets local geodetic coordinates E (East horizontal component) and N (North horizontal component) of 56 points GPS that form the canvas auscultation . The study in two dimensions (2D) allowed us to define the mode of deformation in a horizontal plane (expansion, contraction, total dilation, shear, total rotation and differential rotation).
– Case of a 1D network: The description of the behavior of the local site in the vertical direction (swelling or compaction) is based on the use of the U geodetic coordinate local (vertical component or elevation) of points GPS (56 benchmarks) that constitute auscultation canvas.

5.1 **Horizontal deformation (2D)**
5.1.1 Displacement field
From Figure (5), we note that the displacements of some points in the North-East reservoir (Sea side) are more important than other modules noted in the region. We also find that displacement direction of the most points in network is North-East. The statistics table (1) of displacement, shows that all average displacement network points is the order of 4.4 cm with a maximum displacement ($\cong 18$ cm) located North-East of the reservoir (point P043).

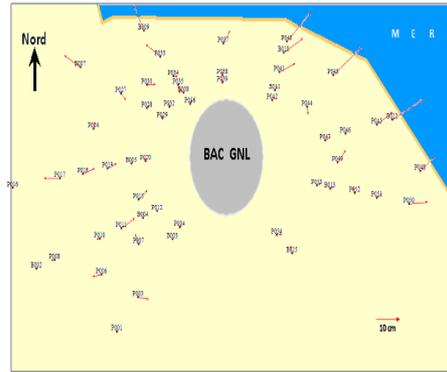|	Displacement (cm)	Corresponding points
Min	0.1	B003
Max	17.7	P043
Moy	4.4	

Table 1: Displacements statistics.

Fig. 5: Displacement field *(2000-2006)*

5.1.2 Deformation tensor

Figure (6) shows that deformation mode of majority triangular elements network, mainly in the North and North-East (Sea side) of the reservoir is an expansion (dominant red).
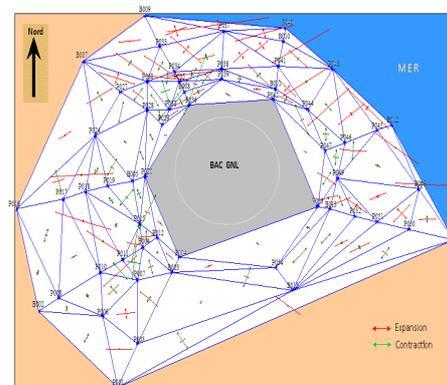

Fig 6: Main constraints (2000-2006).

Table (2) shows maximum, minimum and average values of main constraints (λ_1, λ_2) each network triangle main constraints, between the periods 2000 and 2006. The average amplitude of expansion ($\cong$ 3502 ppm) is higher than that of contraction ($\cong$ 1522 ppm)

		Main Contraintes (ppm)			
		λ_1 (ppm)	**Triangles**	λ_2 (ppm)	**Triangles**
Min		-2631,9 *contraction*	P007, P011, B004	-12833,9 *contraction*	B008, P035, P039
Max		20700,4 *expansion*	P037, B009, P040	2700,8 *expansion*	P050, P051, P048
Moy		3502,3 *expansion*		-1522,0 *contraction*	

Table 2: Main statistics contraints.

5.1.3 Deformations Primitives:

- **Dilatation (Expansion / Contraction):**   Figure (7) shows that deformation in terms of expansion (blue nuance) is on one hand dominant (61%), other part more important in Northern, North-East (Sea side) and South -West (Earth side) of the reservoir. Moreover, we note from Table (3) that the average amplitude of the deformation in expansion terms ($\cong$ 2564 ppm) is greater than that of deformation in contraction terms ($\cong$ 1605ppm). Finally, in the totality of the results considered, the network is much more distorted in expansion than contraction.
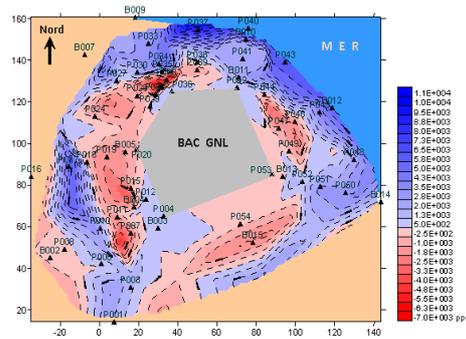

Fig.7: Dilatation *(2000-2006).*

	Dilatation (ppm)			
	Contraction	**Triangles**	**Expansion**	**Triangles**
Min	77,1		58,3	
		P032, P035, B008		P054, P053, B013
Max	6589,7		11480,8	
		P008, P010, P006		P037, B009, P040
Moy	1604,9		2564,2	

Table 3: Dilatation statistics.

- Total shear: Table (4) shows that mean amplitude of shear deformation ($\cong$ 2512ppm) is alike to expansion deformation ($\cong$ 2564ppm) (Table 3).

	Cisaillement Total (ppm)	**Triangles**
Min	208,0	P011, P018, P015
Max	9219,6	P037, B009, P040
Moy	2512,2	

Table 4 : Statistics of Shear.

From Figure (8), we note that highest shear deformations are detected mainly in North, North-East (Sea side), and Southwest (Earth side) areas. We can see that the regions where shear deformations are maximums   broadly correspond to those where deformations in dilation (expansion) are more important (dark blue).
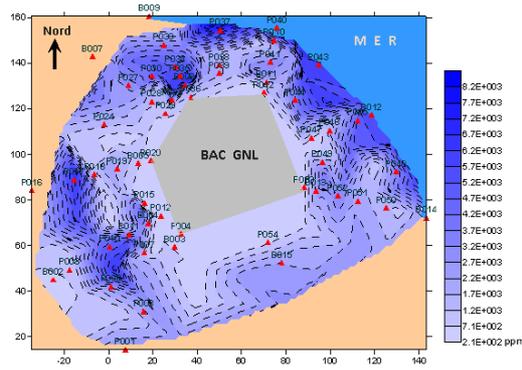
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Fig. 8: Total shear·

- Differential rotation: Figure (9) shows that most network areas (red) had negative rotation (East-West) of average $\cong$ 0°07. Table (5) shows that the average of negative rotation angles ($\cong$ 0°07) is almost identical to the positive rotation angles ($\cong$ 0°08).
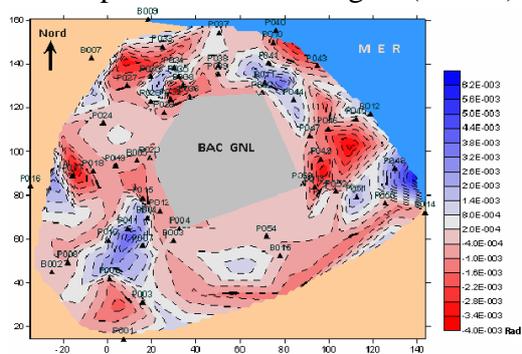

Fig. 9: Differential rotation *(2000-2006)*

Figure (9) shows that most network areas (red) had negative rotation (East-West) of average $\cong$ 0°07. Table (5) shows that average of negative rotation angles is almost identical to the positive rotation angles.

		Rotation Différentielle Propre (degrés)			
		Sens (-)	**Triangles**	**Sens (+)**	**Triangles**
Min		0° 0024	P020, P028, P029	0° 0012	B010, P037,P040
Max		0° 2152	P049, P046, P045	0° 3865	P048, B012,B014
Moy		0° 0670		0° 0792	
Rotation différentielle propre Moyenne du réseau : *+ 0° 0061 (Ouest-Est)*					

Table 5: Differential rotation statistics *(2000-2006)*

5.1.4   <u>Significance of the deformation tensor</u>**:**
The deformation parameters (2D) chosen for the study of deformability is expansion, shear,

and absolute value of the differential rotation. These primitives are scalars amplitudes positives rising with deformations. Figures (8), (9) and (10) show:

- In green areas, significant deformations and in order of deformability (DS&OD).

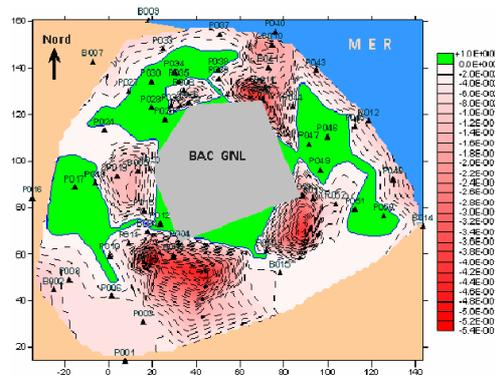- In red regions, the deformations found are not significant (DNS).

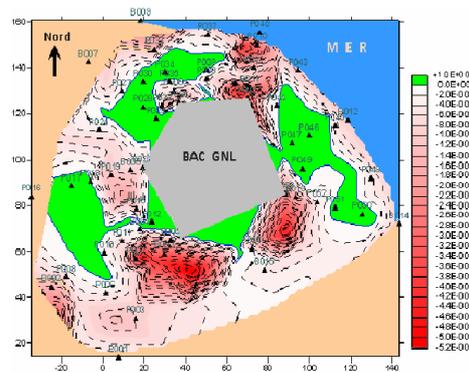**Fig. 10 :** Dilatation significance *(2000-2006).*

Fig. 11 : Shear significance *(2000-2006).*
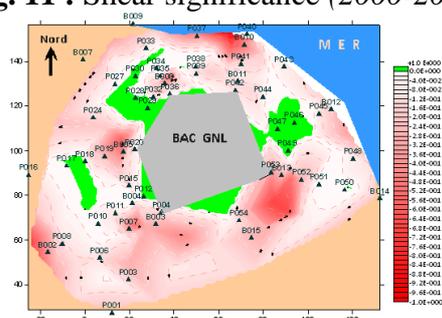
Fig. 12: differential Rotation significance *(2000-2006).*

According to pictures 10, 11 and 12, we note that measured deformations are lower to those that the network can detect (DNS), red dominating. And we can see also that deformation measured in expansion, shear and rotation, described as (DS&OD) is one hand in the most part at the same regions of network (green nuance), mainly in the areas surrounding the

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reservoir (North-East ,South-West) , and the other hand In the same areas where maximum deformations have been detected (figures 7, 8 and 9).   We also note, from Table (6) deformation of all primitives estimated are lower than the network deformability.

	Min	Max	Moy
Dilatation Deformability	491,3	21954,5	5188,8
Dilatation Totale (ppm)	491,3	20823,5	4894,4
Shear Deformability	208,0	9872,5	2674,4
Shear (ppm)	208,0	9219,6	2512,2
Rotation Deformability (°)	0° 002	0° 437	0° 081
Rotation (°)	0° 001	0° 386	0° 079

Tableau 6: Primitives deformation statistics *(2000-2006).*

6.   VERTICAL DEFORMATION (1D)

6.1 Gradient of vertical displacement field :
The gradient of vertical displacement field is used to determine deformation along the vertical component direction (swelling or compaction).The areas affected from Figure (9) by compaction (red) are more numerous   than those the swellings' (blue), approximately (60%). The amplitude of the field gradient of vertical displacement in compaction and swelling at areas South-West reservoir (Earth side) is more important than the areas to the East. However the average amplitude of swelling is greater than that of compaction (Table 7).
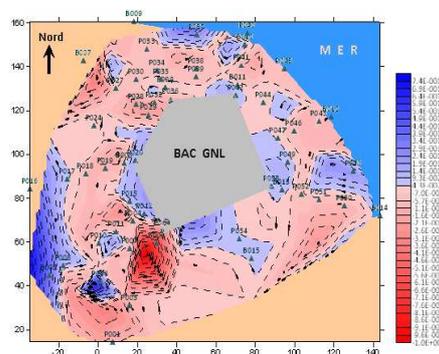

Fig. 13: Gradient vertical displacement field (2000-2006).

	Gradient of vertical displacement field (Unit elongation)			
	Compaction	**Triangles**	**Swelling**	**Triangles**
Min	0,003499	P035, P038, P039	0,000780	P049,P047, P046
Max	1,059294	P054, P004, P053	0,828593	P008, P016, P017
Moy	0,108850		0,147065	

Table 7: Swelling / Compaction   (2000-2006).

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6.2 Significance of vertical displacement field gradient:

The significance of   network estimated deformation in compaction and swelling, Figure (10) shows that most parts of the study area did not   deform significantly (red color dominates the graph). However, it should be noted that there are deformation that can be described as "significant and order of deformability (white area surrounded by a blue line) in South, South-West, North, North-East and North-West of reservoir.
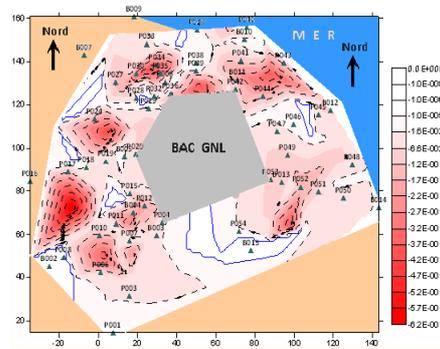

Fig. 14: Significance of vertical displacement field Gradient (2000-2006).

CONCLUSION:

The study of deformation in a geodetic network using the deformation tensors have the advantage of being independent of reference system chosen. The local behavior of a site can be described using tensor formalism that defines several parameters to twofold interest: The first is that they clarify the mode of deformation (for example, the object studied twists and / or dilates) and the second interest is that they are quantifiable (for example, the object dilates himself of a quantity in a given direction).

On the other hand the estimation of deformability, which is an inherent feature of network, determines deformations a priori   that network of geodetic observations will be able to detect. However definition of significance fits properly with deformation monitoring of geodetic networks. In hypothesis, where observed networks stay similar (same measuring points, same measures, and same observations accuracy). However, if this hypothesis is not verified, it is essential to consider the case of networks observed differently. Deformation analysis results differ depending on the choice of elementary figures [Michel. K].The contribution of the Delaunay triangulation resides in the fact that it permits by the criteria of the uniqueness of the triangulation, the uniqueness of the deformation analysis. However, this method is more appropriate to good and dense geodetic networks geometry. In the case of geodetic networks of low density, it is necessary to use other tools interpolation of field displacement to remedy the insufficient density and ensure continuous information. Indeed, no method of this type can create more information than there is under form of geodetic points measured (real). To this effect, for geodetic networks of small extent (case of our application), it is wise to achieve a canvas of geodetic points uniformly distributed, leading to a regular mesh where nodes are consequent the geodetic measurements.

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