Rigid Plate Transformations to Support PPP and Absolute Positioning in Africa

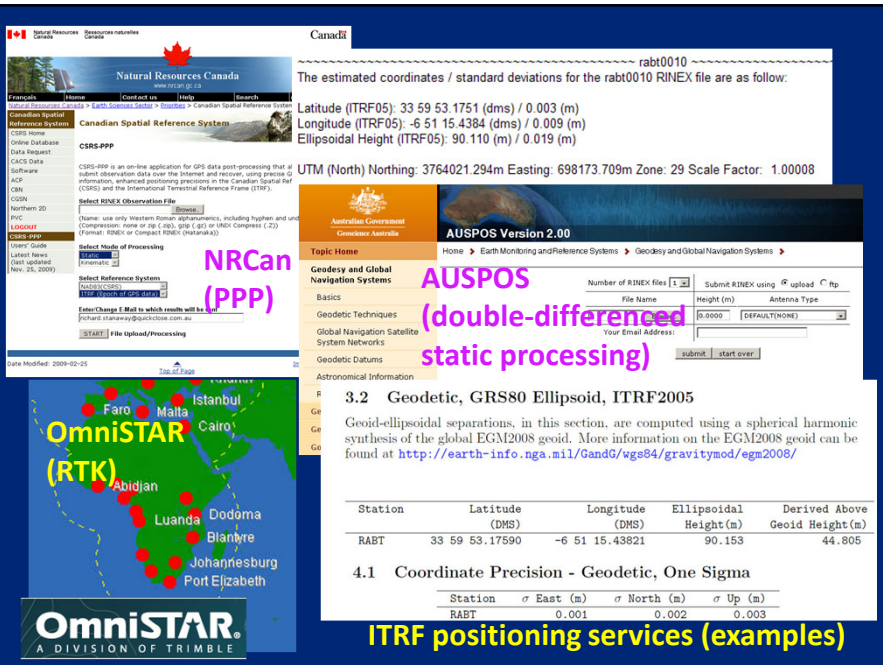
Richard Stanaway & Craig Roberts
School of Surveying and Spatial Information Systems
University of New South Wales, Australia

FIG Working Week, Marrakech, Morocco, 18-22 May 2011

CORS Distribution in Africa

Sparse GNSS CORS infrastructure overcome by use of PPP and Global Differential Services

image: Centro GNSS de Canarias
www.canarygnsscenter.org



ITRF positioning services (examples)

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Kinematic coordinates illustrated

← ITRF2008 Epoch 2008
N 6° 52' 14.0169" W 5° 14' 24.3345"

YKRO – IGS Station
(Yamoussoukro, Cote d' Ivorie)
From IGS web-site

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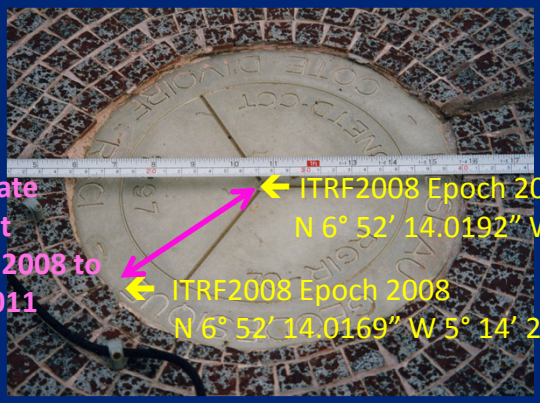
Kinematic coordinates illustrated



Nubian Plate movement 1 January 2008 to 22 May 2011 109 mm!

ITRF2008 Epoch 2011.4
N 6° 52' 14.0192" W 5° 14' 24.3317"

ITRF2008 Epoch 2008
N 6° 52' 14.0169" W 5° 14' 24.3345"

YKRO – IGS Station
(Yamoussoukro, Cote d' Ivorie)
From IGS web-site

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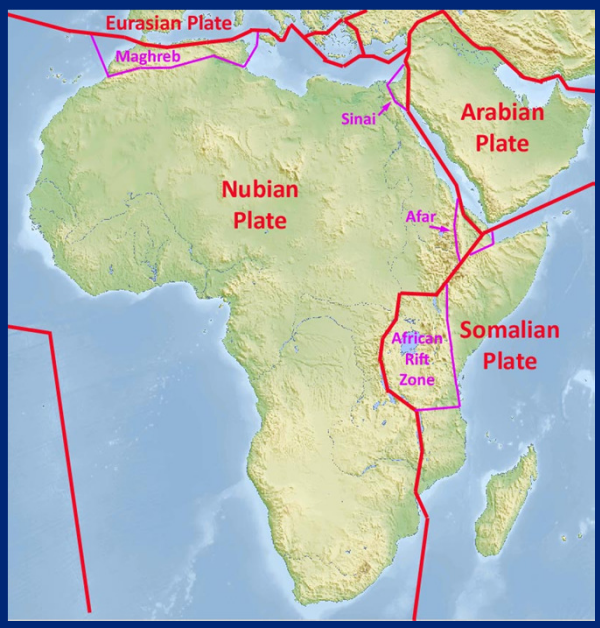

Principal Plates and Plate Boundaries in Africa

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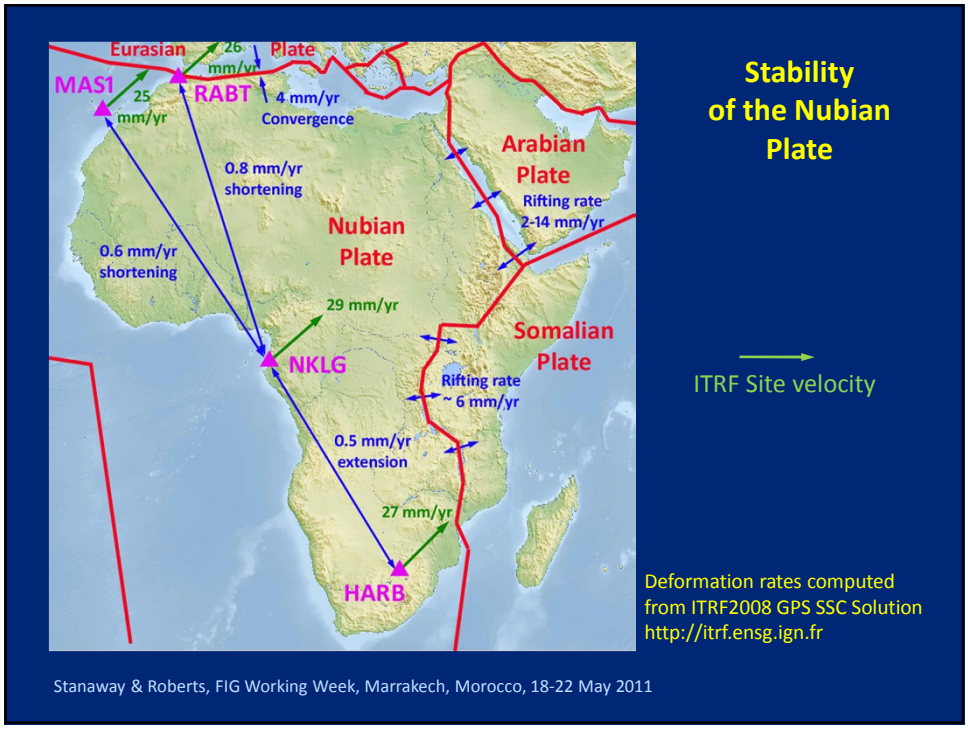
Eurasian Plate
26 mm/yr
MAS1
25 mm/yr
RABT
4 mm/yr Convergence
0.8 mm/yr shortening
Arabian Plate
Rifting rate 2-14 mm/yr
Nubian Plate
0.6 mm/yr shortening
29 mm/yr
Somalian Plate
NKLG
Rifting rate ~ 6 mm/yr
0.5 mm/yr extension
27 mm/yr
HARB
Stability of the Nubian Plate
ITRF Site velocity
Deformation rates computed from ITRF2008 GPS SSC Solution
http://itrf.ensg.ign.fr
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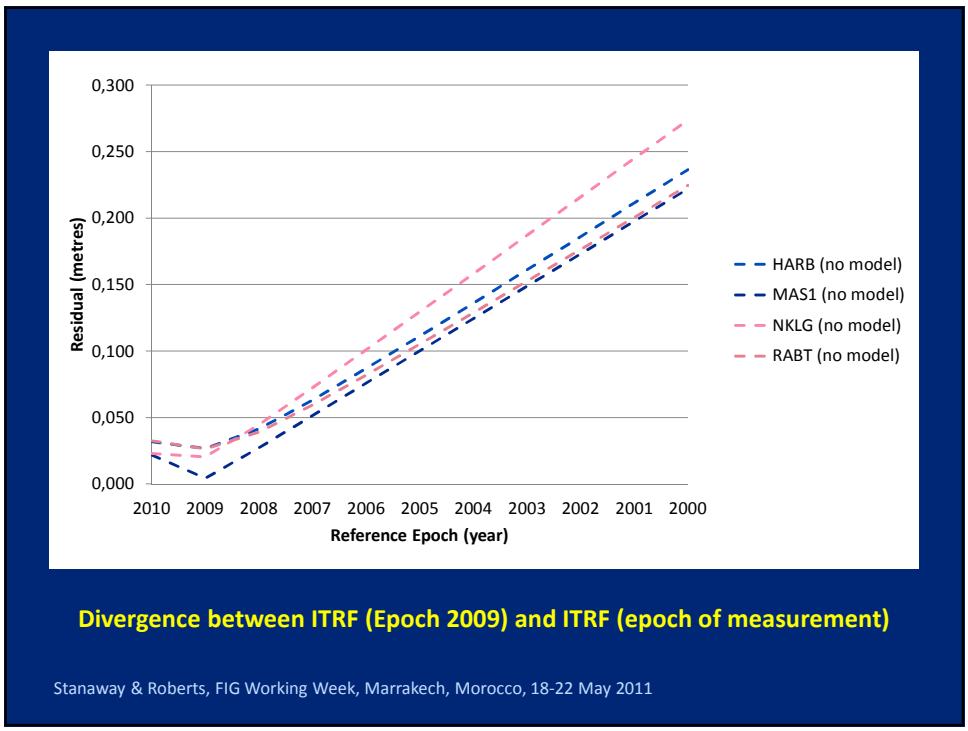
0,300
0,250
0,200
0,150
0,100
0,050
0,000
Residual (metres)
2010 2009 2008 2007 2006 2005 2004 2003 2002 2001 2000
Reference Epoch (year)
HARB (no model)
MAS1 (no model)
NKLG (no model)
RABT (no model)
Divergence between ITRF (Epoch 2009) and ITRF (epoch of measurement)
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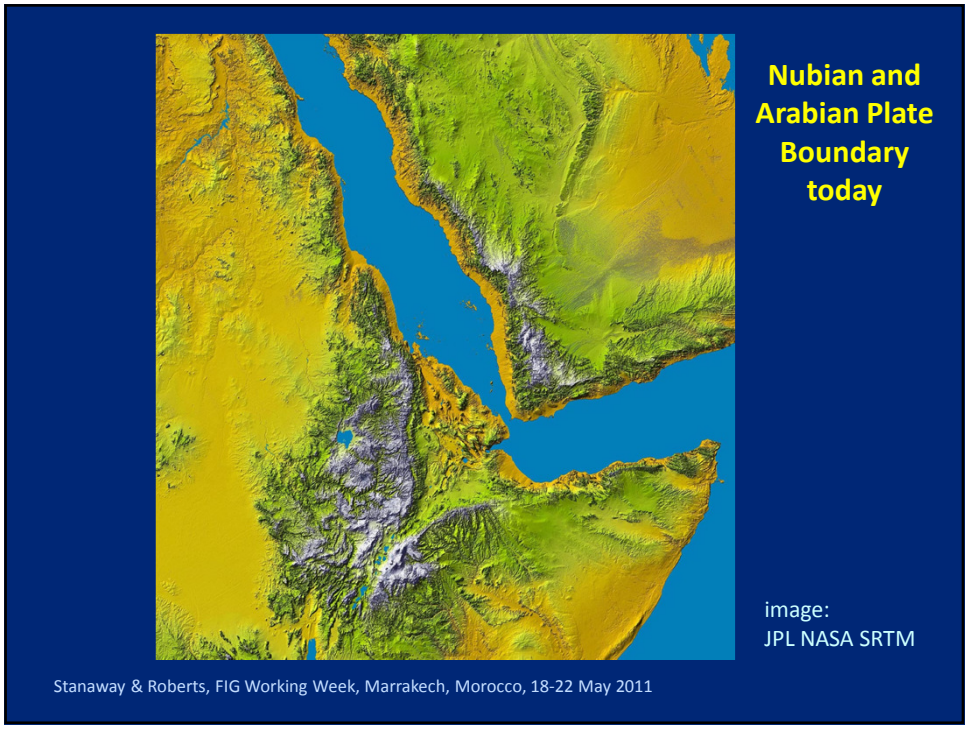
Nubian and Arabian Plate Boundary today
image:
JPL NASA SRTM
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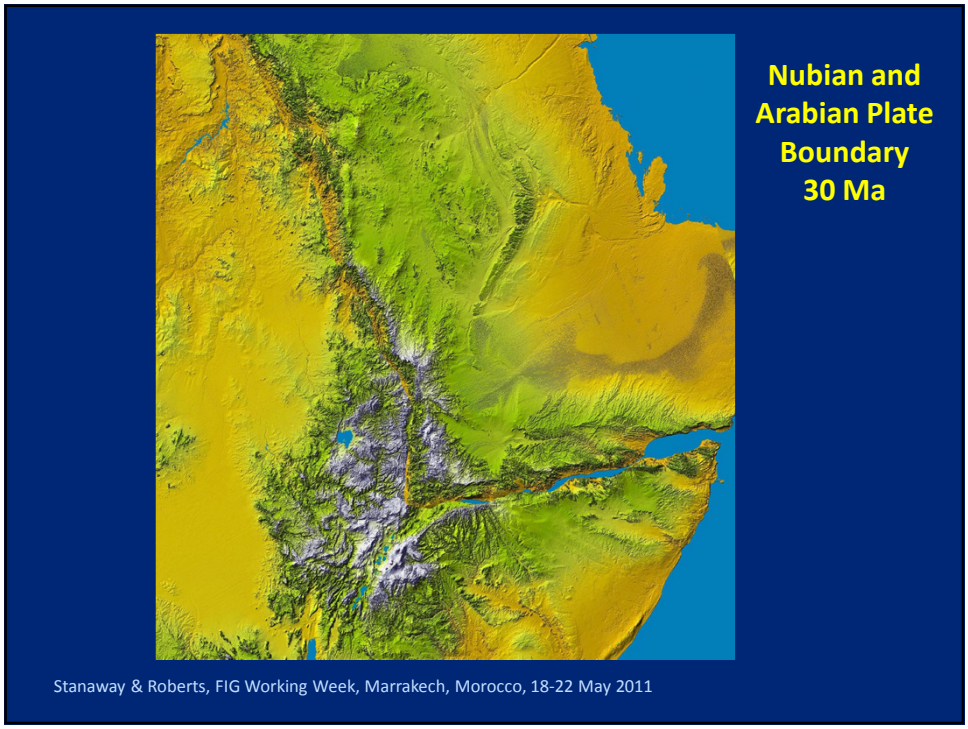
Nubian and Arabian Plate Boundary 30 Ma
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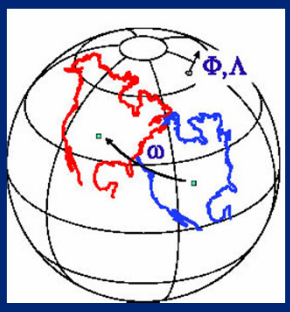
$\Omega_X = \mathrm{COS}(\Phi)\mathrm{COS}(\Lambda)\omega$

Rigid Plate Model



Euler Poles to Cartesian rotation rates

$$\Omega_X = \mathrm{COS}(\Phi)\mathrm{COS}(\Lambda)\omega$$
$$\Omega_Y = \mathrm{COS}(\Phi)\mathrm{SIN}(\Lambda)\omega$$
$$\Omega_Z = \mathrm{SIN}(\Phi)\omega$$

ITRF2005 African plate parameters (Altamimi *et al.* 2007)

Plate	Euler pole of rotation			Equivalent Cartesian angular velocity		
	Φ (°)	Λ (°)	ω (°/Ma)	Ω_X (Rad/Ma)	Ω_Y (Rad/Ma)	Ω_Z (Rad/Ma)
Arabia	49.6	5.1	0.579	0.006518	0.000577	0.007700
Eurasia	56.3	-96.0	0.261	-0.000263	-0.002512	0.003791
Nubia	50.0	-82.5	0.269	0.000394	-0.002995	0.003594
Somalia	53.7	-89.5	0.309	0.000026	-0.003196	0.004344

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Kinematic to Static transformation

$$\begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} + S \cdot \left(\begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix} + \begin{bmatrix} \Omega_Y Z_t - \Omega_Z Y_t \\ \Omega_Z X_t - \Omega_X Z_t \\ \Omega_X Y_t - \Omega_Y X_t \end{bmatrix} \cdot (t_0 - t) \cdot 1\text{E-}6 \right)$$

"Static" coordinates at reference epoch

Local frame translation & scale (only if required)

"Measured" ITRF coordinates

Plate rotation parameters

reference epoch

measurement epoch

$$X_0 = X_t + (\Omega_Y Z_t - \Omega_Z Y_t).(t_0 - t) \cdot 1\text{E-}6$$
$$Y_0 = Y_t + (\Omega_Z X_t - \Omega_X Z_t).(t_0 - t) \cdot 1\text{E-}6$$
$$Z_0 = Z_t + (\Omega_X Y_t - \Omega_Y X_t).(t_0 - t) \cdot 1\text{E-}6$$

Simplified 3-parameter equations Kinematic ITRF to Static ITRF
(no scale or translation parameters)

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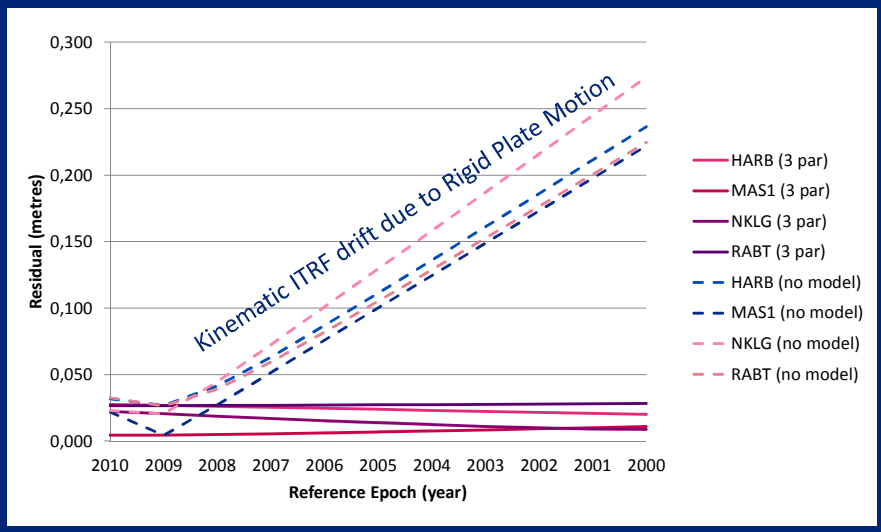

Improved coordinate consistency using a 3 parameter rigid plate transformation

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Limitations of a Rigid Plate Model

- Intraplate deformation not accounted for (usually small magnitude < 1 mm/yr anyway)
- Fails near plate boundaries (requires additional modelling of locked faults)
- Coseismic and Postseismic deformation not modelled

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Thank you
Merci
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