Geodetic or Rhumb Line Polygon Area Calculation over the WGS-84 Datum Ellipsoid

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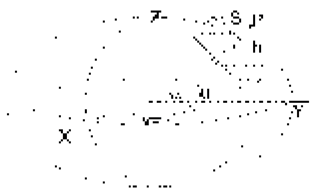

Figure 01 – Geodetic and Geocentric Latitudes and Longitudes,

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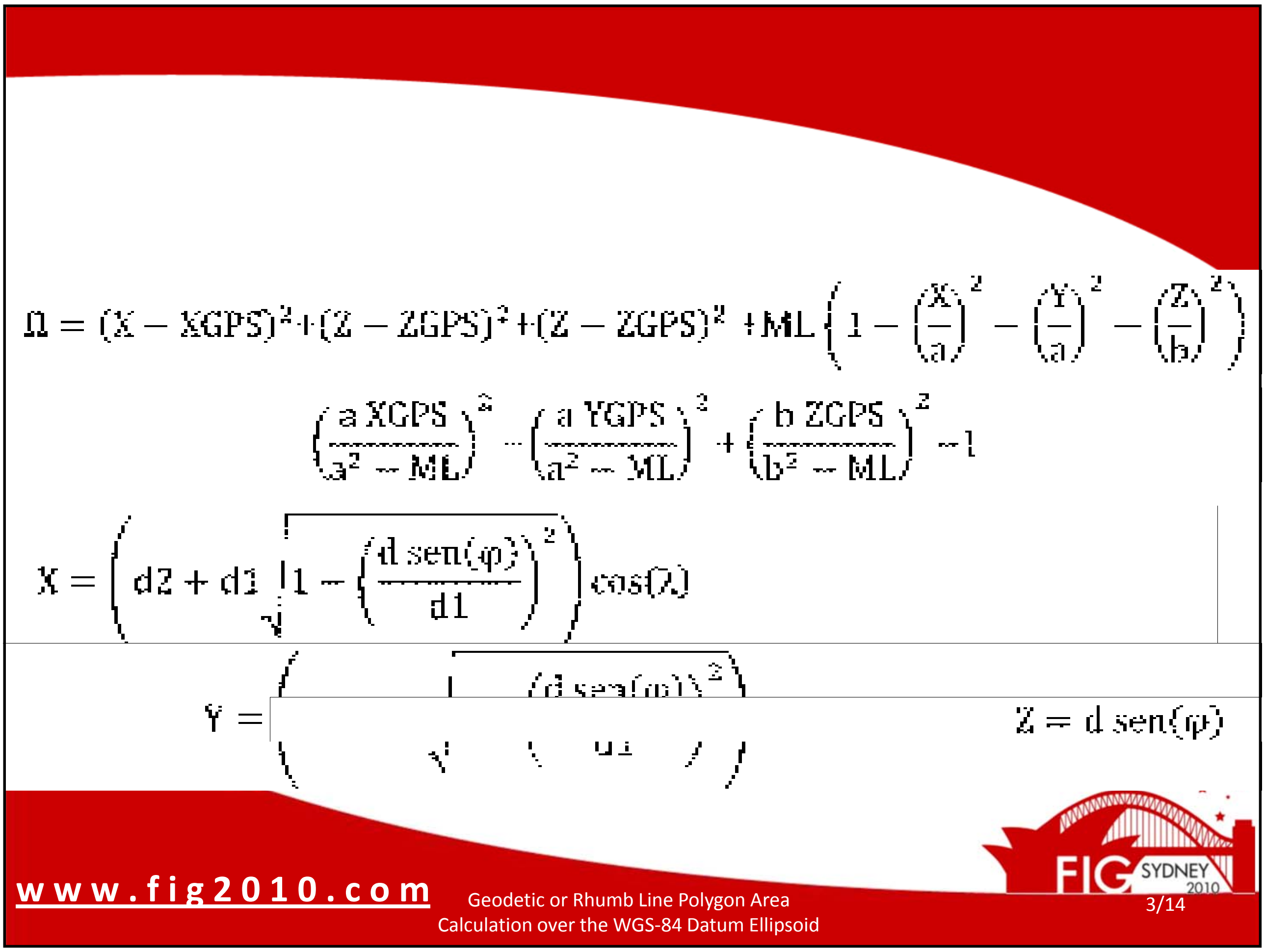
$\Omega = (X - XGPS)^2 + (Z - ZGPS)^2 + (Z - ZGPS)^2 + ML\left(1 - \left(\frac{X}{a}\right)^2 - \left(\frac{Y}{a}\right)^2 - \left(\frac{Z}{b}\right)^2\right)$
$\left(\frac{a\,XGPS}{a^2 - ML}\right)^2 + \left(\frac{a\,YGPS}{a^2 - ML}\right)^2 + \left(\frac{b\,ZGPS}{b^2 - ML}\right)^2 - 1$
$X = \left(d2 + d1\sqrt{1 - \left(\frac{d\,sen(\varphi)}{d1}\right)^2}\right)\cos(\lambda)$
$Y =$
$Z = d\,sen(\varphi)$
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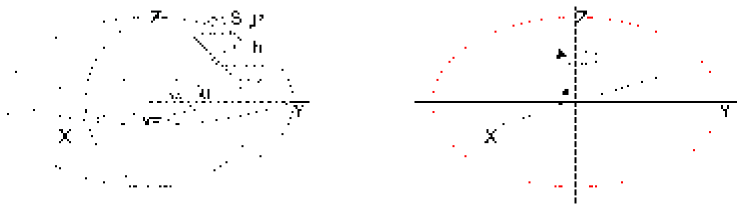
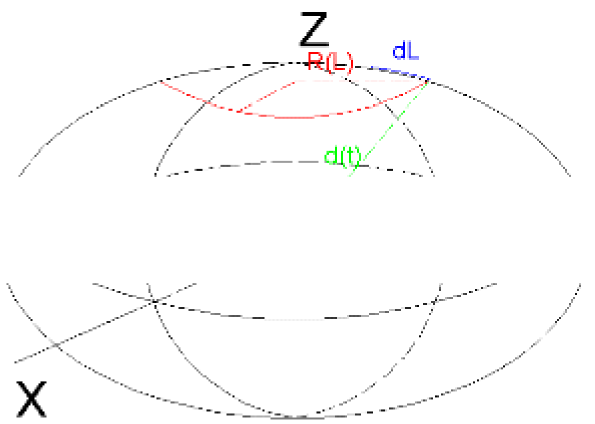


Figure 01 – Geodetic and Geocentric Latitudes and Longitudes, line integration and Pappus method,



$$\int 2\pi R(L)\, dL$$

$$dL = \sqrt{\left(\cos(t)\frac{d}{dt}(d(t)) - d(t)\operatorname{sen}(t)\right)^2 + \left(\operatorname{sen}(t)\frac{d}{dt}(d(t)) + d(t)\cos(t)\right)^2}\, dt$$

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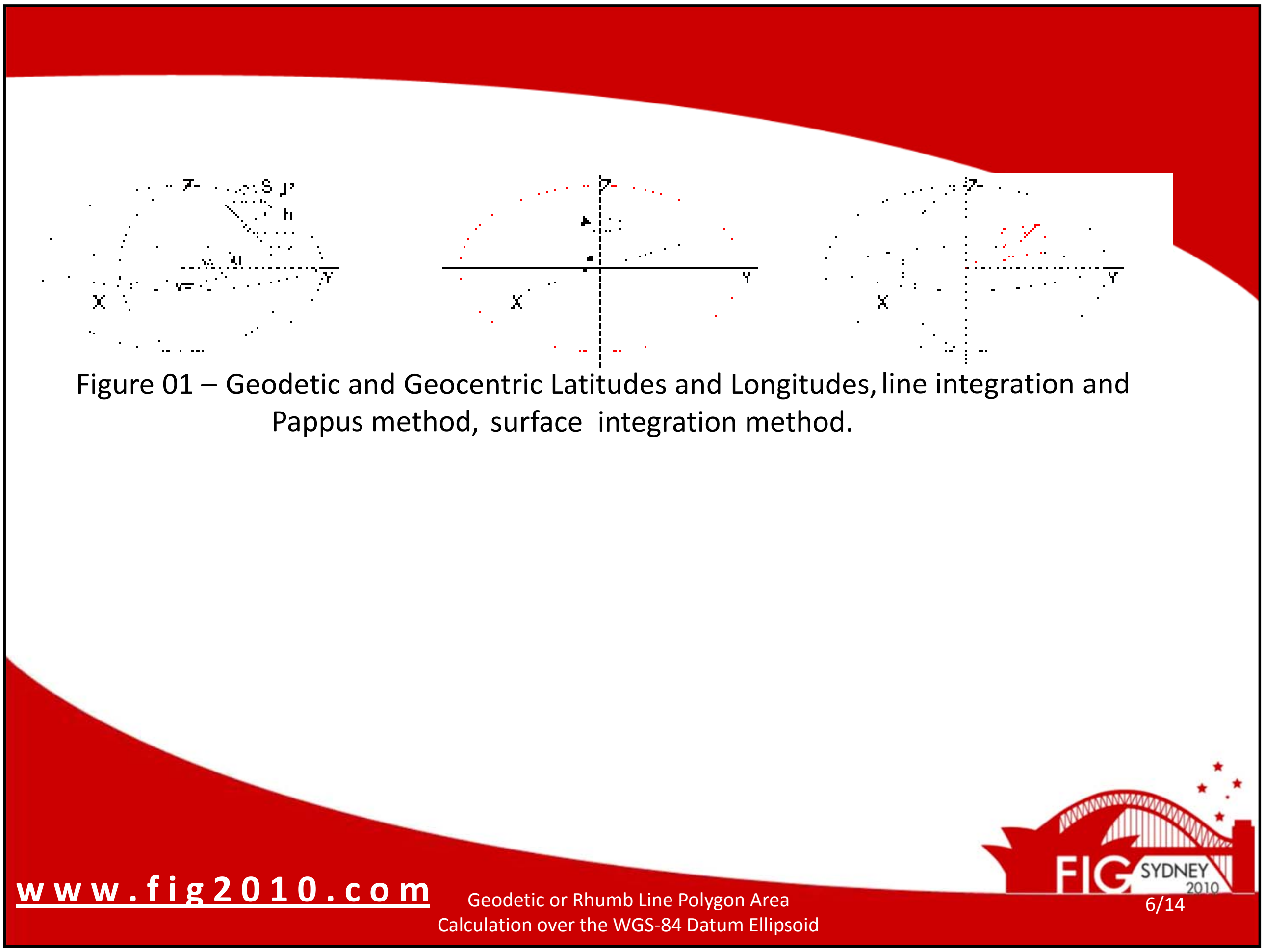
Figure 01 – Geodetic and Geocentric Latitudes and Longitudes, line integration and Pappus method, surface integration method.
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$$\iint \left\| \frac{\partial}{\partial \varphi} r(\varphi, \lambda) \times \frac{\partial}{\partial \lambda} r(\varphi, \lambda) \right\| d\varphi d\lambda$$

$$8 \int_{0^0}^{90^0} \int_{-90^0}^{0^0} \frac{d^4 \cos(\varphi) \sqrt{b^4 \cos^2(\varphi) - a^4 \mathrm{sen}^2(\varphi)}}{(a\,b)^2} d\varphi d\lambda$$

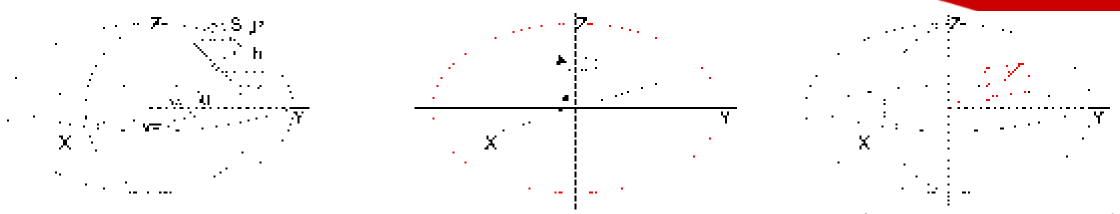

Figure 01 – Geodetic and Geocentric Latitudes and Longitudes, line integration and Pappus method, surface integration method.

Total area: 510.065.621.724.088,44 m^2

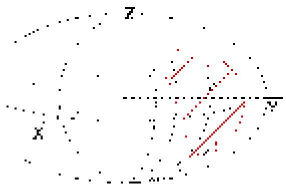

Figure 02 –Gauss Polygon Area Adaptation,

$$\int_{\lambda 1}^{\lambda 2}\int_{-90^0}^{\varphi(\lambda)} \frac{d^4\cos(\varphi)\sqrt{b^4\cos^2(\varphi)+a^4 sen^2(\varphi)}}{(a\,b)^2}\, d\varphi d\lambda$$

$$\int_{\lambda 1}^{\lambda 2}\int_{-90^0}^{\varphi(\lambda)}\sum_{k=0}^{n}\left(a_k\,\varphi^k\right) d\varphi d\lambda$$

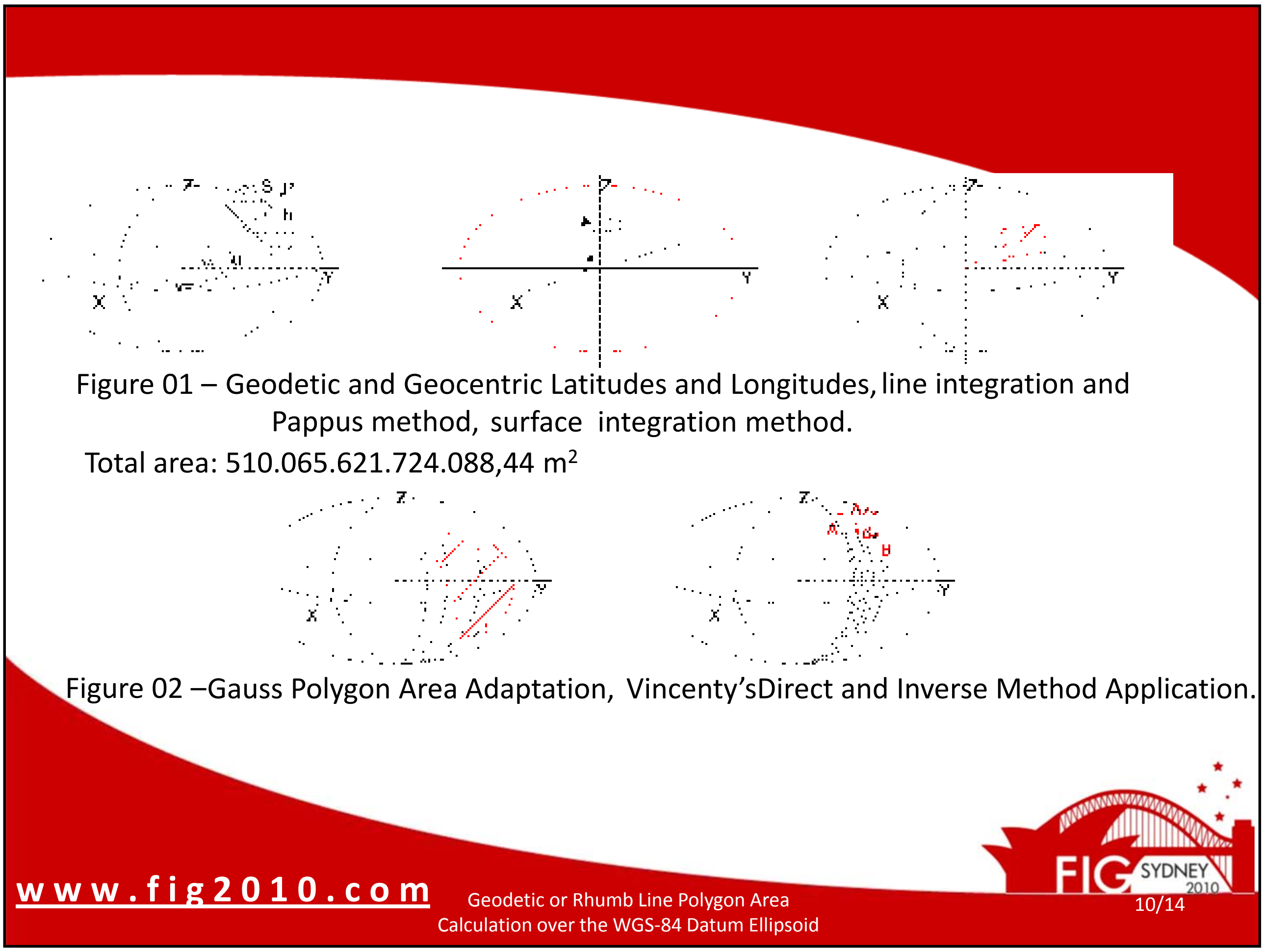
Figure 01 – Geodetic and Geocentric Latitudes and Longitudes, line integration and Pappus method, surface integration method.
Total area: 510.065.621.724.088,44 m²
Figure 02 –Gauss Polygon Area Adaptation, Vincenty'sDirect and Inverse Method Application.
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$$\int_{\lambda 1}^{\lambda 2}\int_{-90^{\circ}}^{\operatorname{atan}\left(\frac{-coef1\cos(\lambda)-coef2\,sen(\lambda)}{coef3}\right)}\sum_{k=0}^{n}\left(a_k\,\varphi^k\right)d\varphi\, d\lambda$$

$$\int_{\lambda 1}^{\lambda 2}\sum_{k=0}^{n}\left\{\frac{a_k}{k+1}\left[\operatorname{atan}\left(\frac{-coef1\cos(\lambda)-coef2\,sen(\lambda)}{coef3}\right)\right]^{k+1}\right\}d\lambda$$

$$\sum\int_{\lambda 1}^{\lambda 2}\sum_{k=0}^{n}\left\{\frac{a_k}{k+1}\left[\operatorname{atan}\left(\frac{-coef1\cos(\lambda)-coef2\,sen(\lambda)}{coef3}\right)\right]^{k+1}\right\}d\lambda$$

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Geodetic Polygon Area Results:

a) Half WGS-84 ellipsoid: 38ppq;

b) Against CARIS solution 5 polygonals, 4.764,5km^2 to 47.187.272,2km^2: 7,7ppm to 3,8ppb;

c) Against 11° semi-fuse: 0,3ppm (CARIS) 3,4ppq (AreaCalc);

d) Small polygonal area (123,09m^2): 95,2ppt.

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Rhumb Lines Polygon:

a) Same methodology as geodetic approach;
b) Instead Vincenty, Mercator Projection;
c) Results were similar to geodetic ones.

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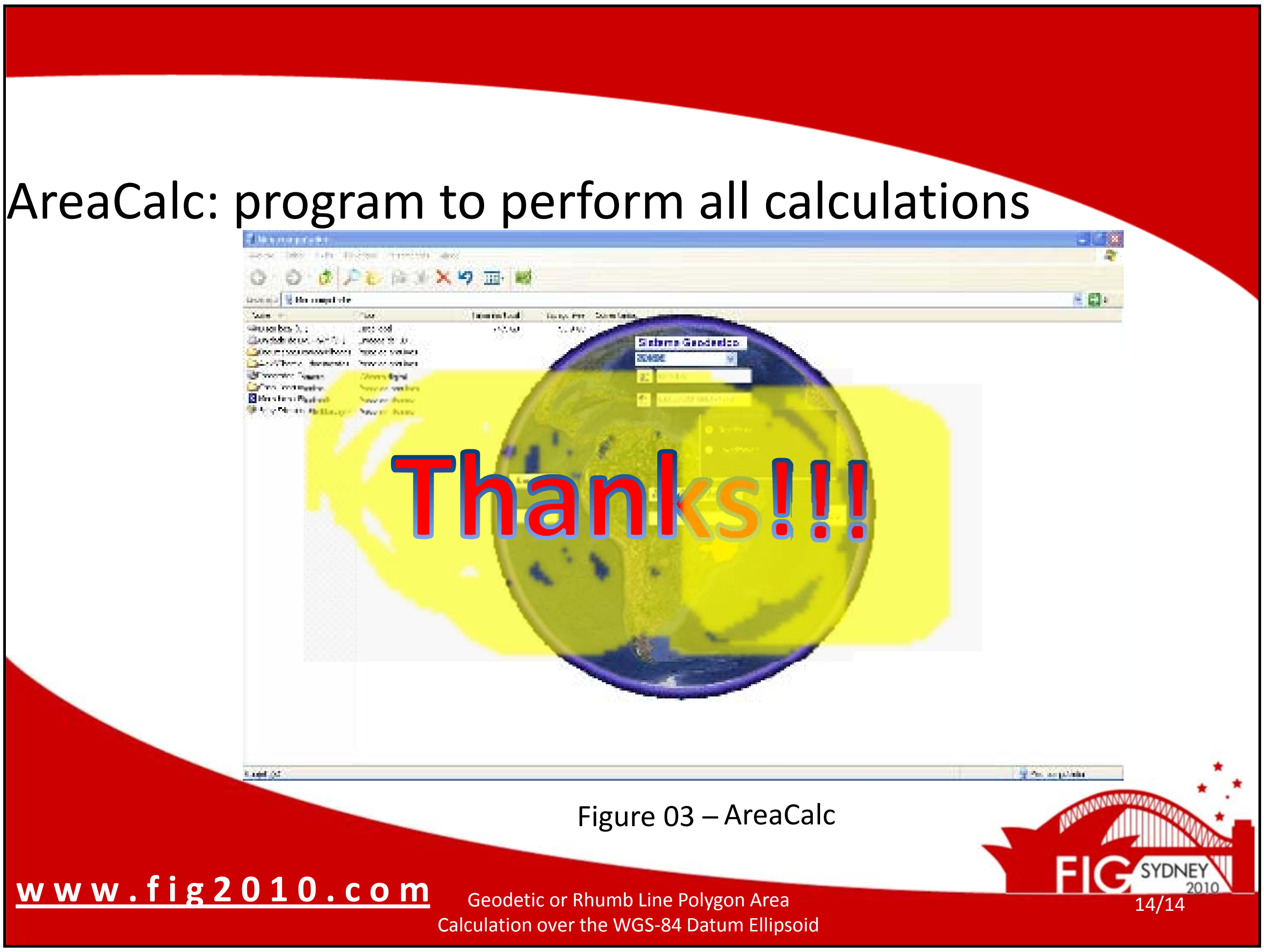
AreaCalc: program to perform all calculations
Thanks!!!
Figure 03 – AreaCalc
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