"THE CONCEPTION OF MONITORING THE SUPERFICIAL DEFORMATION LOCATED ON THE UNSTABLE FOUNDATION WITH THE USAGE OF GPS TECHNOLOGY"

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Abstract: In this paper a new method for monitoring surface deformations is presented. Theoretical consideration are supplemented an example of practical application.

1. Introduction

The deformation measurement, including the earth surface deformation, is present problem in the modern engineering geodesy. The monitoring of deformation located on the unstable foundation and assigned in unstable reference system is especially important problem. The unstable foundation is defined as surface that can be influenced by movements together with the engineering objects located on it. The problem of deformation assigning is still present because a lot of building investments are situated at city centres, next to existing buildings. The earth works aiming at creation of for example underground multi-storey car parks, can cause the movements of area surrounding the building area (including the local reference system) and at the same time can influence on the existing buildings causing the strains and mutual displacements.

The author of one paper (Wiśniewski 1989) proposed a concept of determining the parameters of position and shape of foundation plates.

In this paper the conception of monitoring earth surface deformation located on the unstable foundation with usage of GPS technology is presented (more information in paper Kamiński 2008). The formulas solving the problem including the accuracy analysis defined by the covariance matrix are presented. The empirical tests were made. The interpretation of interesting results coming from the tests encourages to conduct further and more detailed theoretical and empirical analyses.

2. The Concept Development

Let us assume that, as is frequently the case in engineering practice, the reference surface is plane Ω, whose fragment in any local co-ordinate system (X,Y,Z), is shown in Fig.1.

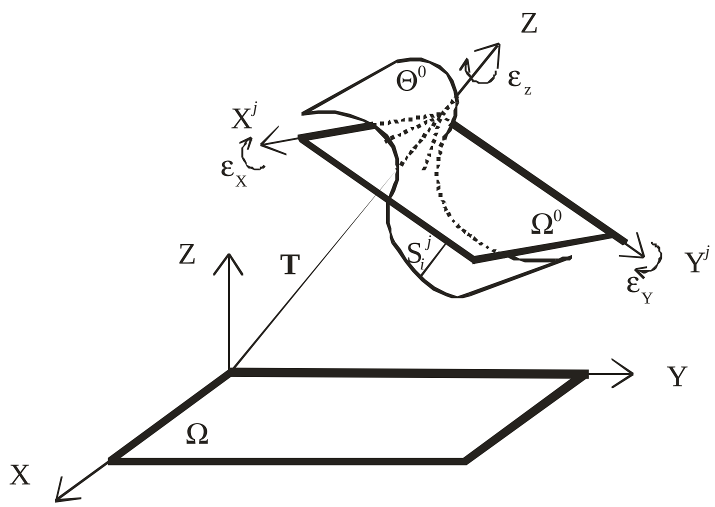

Figure1

Let us further assume that after conducting an initial measurement at time moment $t^{j=0}$ (j=0,1,2,3,…) we obtained a surface $\Theta^{j=0}$, whose representation is plane $\Omega^{j=0}$. The plane Ω^0 is defined in an instantaneous reference system ($X^{j=0}$,$Y^{j=0}$,$Z^{j=0}$). Performing control measurements at time moments t^1, t^2,t^3,…, we get, respectively, surfaces Θ^j and planes Ω^j, (j=1,2,3,…).

In figure 1 s_i^j denotes the distance between an irregular surface Θ^j, resulting, e.g. from uneven elevation (settlement) of points and the optimum plane Ω^j at any point i, j =0,1,2,3,…- the moment of measurement. Distances s_i^j determined by the least squares method, can be treated as the length of vector $\mathbf{s}^j = [s_X^j, s_Y^j, s_Z^j]^T$, ($s_X^j, s_Y^j, s_Z^j$ - components of the vector), which is perpendicular to plane Ω^j and which determines instantaneous positions of the plane in an unstable reference system X^j, Y^j, Z^j.

In order to carry out further specialist analyses, the coordinates of a point determined in the measurements, i.e. in the X^j, Y^j, Z^j system, should then by recalculated to the X,Y,Z reference system. The coordinates can be recalculated with the use of a 6-parameter transformation, with the following parameters:

- $\mathbf{T}=[T_X,T_Y,T_Z]^T$ translation vector of the centre of the reference system,

- $\mathbf{R}=[\varepsilon_X, \varepsilon_Y, \varepsilon_Z]^T$ vector of the angles of rotation around the axis of the reference system X^j, Y^j, Z^j.

The 6-parameter transformation is usually carried out according to the following principle:

$$\mathbf{X}_i = \mathbf{T}_i^j + \mathbf{R}^j \mathbf{x}_i^j \tag{2.1}$$

The relationship $\mathbf{x}_i^j = \left[x_i^j \; y_i^j \; z_i^j\right]^T$ denotes a vector of coordinates of the i-th point in an unstable instantaneous reference system of the j-th current measurement.

Formula (2.1) yields the following equations (for an individual point):

$$\begin{aligned} X_i &= T_X^j + x_i^j + \varepsilon_Z^j y_i^j - \varepsilon_Y^j z_i^j \\ Y_i &= T_Y^j - \varepsilon_Z^j x_i^j + y_i^j + \varepsilon_X^j z_i^j \\ Z_i &= T_Z^j + \varepsilon_Y^j x_i^j - \varepsilon_X^j y_i^j + z_i^j \end{aligned} \tag{2.2}$$

Using GPS technology in the geodetic measurements whose aim is to determine surface deformations, equations of corrections between points “i” and “k” can be shown in the following form

$$\begin{aligned} v_{\Delta x} &= X_k - X_i - \Delta x_{i,k}^{obs} \\ v_{\Delta y} &= Y_k - Y_i - \Delta y_{i,k}^{obs} \\ v_{\Delta z} &= Z_k - Z_i - \Delta z_{i,k}^{obs} \end{aligned} \tag{2.3}$$

Carrying out the transformation for a pair of points (i, k), the relationship (2.1) should be inserted into the relationship (2.3). Hence

$$\begin{aligned} v_{\Delta x} &= \varepsilon_Z^j \Delta y_{i,k}^j - \varepsilon_Y^j \Delta z_{i,k}^j + \Delta x_{i,k}^j - \Delta x_{i,k}^{obs} \\ v_{\Delta y} &= -\varepsilon_Z^j \Delta x_{i,k}^j + \varepsilon_X^j \Delta z_{i,k}^j + \Delta y_{i,k}^j - \Delta y_{i,k}^{obs} \\ v_{\Delta z} &= \varepsilon_Y^j \Delta x_{i,k}^j - \varepsilon_X^j \Delta y_{i,k}^j + \Delta z_{i.k}^j - \Delta z_{i,k}^{obs} \end{aligned} \tag{2.4}$$

It should be pointed out that such substitution results in the disappearance of the translation vector of the centre of the instantaneous system $\mathbf{T} = \left[\mathrm{T}_X^j, \mathrm{T}_Y^j, \mathrm{T}_Z^j\right]^T$. With the results of measurements made in various epochs, on unstable ground, one cannot determine a vector of translation between unstable reference systems which determine the optimum plane.

Let us consider the difference in coordinates, seen in the penultimate column of relationships (2.4), determined for any pairs of points as $\Delta x_{i,k}^{j} = X_k^j - X_i^j, \Delta y_{i,k}^{j} = Y_k^j - Y_i^j, \Delta z_{i,k}^{j} = Z_k^j - Z_i^j$. It appears from the theoretical assumptions adopted earlier that relationship $\Delta z_{i,k}^{j} = Z_k^j - Z_i^j$, in which Z_k^j and Z_i^j denote the altitudes of points, determines simultaneously distances $s_{Z_k}^j$ and $s_{Z_i}^j$ of the instantaneous surface Θ^j from the instantaneous optimum plane Ω^j. We can also write that $X_i^j = s_{Xi}^j$, and $Y_i^j = s_{Yi}^j$.

As measurements are carried out at various moments in time, the following differences should be considered further

$$\begin{aligned} \Delta \mathbf{s}^{(j+1)-j} &= \mathbf{s}^{j+1} - \mathbf{s}^{j} \\ \Delta \boldsymbol{\varepsilon}^{(j+1)-j} &= \boldsymbol{\varepsilon}^{j+1} - \boldsymbol{\varepsilon}^{j} \end{aligned} \tag{2.5}$$

where $\mathbf{s}^j = [\mathrm{s}_{X_i}^j, \mathrm{s}_{Y_i}^j, \mathrm{s}_{Z_i}^j]$ and $\Delta\mathbf{s}^j = [\Delta \mathrm{s}_{X_{i.k}}^j, \Delta \mathrm{s}_{Y_{i,k}}^j, \Delta \mathrm{s}_{Z_{i,k}}^j]$, $(\Delta s_{X_{i,k}}^j = s_{X_k}^j - s_{X_i}^j$, $\Delta s_{Y_{i,k}}^j = s_{Y_k}^j - s_{Y_i}^j$, $\Delta s_{Z_{i,k}}^j = s_{Z_k}^j - s_{Z_i}^j)$.

The differences shown in the relationship (2.5) can be treated as displacements ($\Delta\boldsymbol{\varepsilon}^{(j+1)-j}$) represented only by angles of rotation $\varepsilon_X, \varepsilon_Y, \varepsilon_Z$ and strains of the object ($\Delta\mathbf{s}^{(j+1)-j}$).

Assuming that angles of rotation $\varepsilon_X, \varepsilon_Y, \varepsilon_Z$ in particular measurement epochs will have low values, the following can be written

$$\begin{aligned} X_i^j &\cong X_i \\ Y_i^j &\cong Y_i \\ Z_i^j &\cong Z_i \end{aligned} \tag{2.6}$$

and, respectively, for coordinate differences

$$\begin{aligned} X_k^j - X_i^j &\cong X_k - X_i \\ Y_k^j - Y_i^j &\cong Y_k - Y_i \\ Z_k^j - Z_i^j &\cong Z_k - Z_i \end{aligned} \tag{2.7}$$

Hence, equation (2.8) can also be written in the following form

$$\begin{aligned}
v_{\Delta x} &= s^j_{Xk} - s^j_{Xi} + (Y_k - Y_i)\varepsilon^j_Z - (Z_k - Z_i)\varepsilon^j_Y - \Delta x^{obs}_{i,k} \\
v_{\Delta y} &= s^j_{Yk} - s^j_{ki} - (X_k - X_i)\varepsilon^j_Z + (Z_k - Z_i)\varepsilon^j_X - \Delta y^{obs}_{i,k} \\
v_{\Delta z} &= s^j_{Zk} - s^j_{Zi} + (X_k - X_i)\varepsilon^j_Y - (Y_k - Y_i)\varepsilon^j_X - \Delta z^{obs}_{i,k}
\end{aligned} \tag{2.8}$$

or in the matrix notation

$$\mathbf{v} = \mathbf{A}_1\mathbf{s}^j + \mathbf{A}_2\boldsymbol{\varepsilon}^j - \mathbf{l} \tag{2.9}$$

in which: **v-** vector of corrections, $\mathbf{A}_1$, $\mathbf{A}_2$ – known matrices of coefficients, $\mathbf{l} = \left[\Delta\mathbf{x}^{obs}, \Delta\mathbf{y}^{obs}, \Delta\mathbf{z}^{obs}\right]^T$ - vector of free terms, **s** and **ε** - estimated values. Because there are no reference points in the method, free adjustment should be employed in the solution of the problem.

3. Adjustment of the Observation Results

The unknowns can be determined by the least squares method. The adjustment problem has the following form

$$\left.\begin{aligned}
\varphi(\mathbf{s}^j, \boldsymbol{\varepsilon}^j) &= \mathbf{v}^T\mathbf{P}\mathbf{v} = \min \\
\mathbf{v} &= \mathbf{A}_1\mathbf{s}^j + \mathbf{A}_2\boldsymbol{\varepsilon}^j - \mathbf{l}
\end{aligned}\right\} \tag{3.1}$$

where **P** is a matrix of weights of the measurement results.

Seeking the minimum of the objective function $\varphi(\mathbf{s}^j, \boldsymbol{\varepsilon}^j) = \mathbf{v}^T\mathbf{P}\mathbf{v}$ with reference to unknowns $\mathbf{s}^j$ and $\boldsymbol{\varepsilon}^j$ yields the following relationships

$$\left.\begin{aligned}
\frac{\partial\varphi}{\partial\mathbf{s}^j} &= 2\mathbf{v}^T\mathbf{P}\mathbf{A}_1 = \mathbf{0} \\
\frac{\partial\varphi}{\partial\boldsymbol{\varepsilon}^j} &= 2\mathbf{v}^T\mathbf{P}\mathbf{A}_2 = \mathbf{0}
\end{aligned}\right\} \tag{3.2}$$

Hence,

$$\left.\begin{aligned}
\mathbf{A}_1^T\mathbf{P}\mathbf{A}_1\mathbf{s}^j + \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2\boldsymbol{\varepsilon}^j - \mathbf{A}_1^T\mathbf{P}\mathbf{l} = \mathbf{0} \\
\mathbf{A}_2^T\mathbf{P}\mathbf{A}_1\mathbf{s}^j + \mathbf{A}_2^T\mathbf{P}\mathbf{A}_2\boldsymbol{\varepsilon}^j - \mathbf{A}_2^T\mathbf{P}\mathbf{l} = \mathbf{0}
\end{aligned}\right\} \tag{3.3}$$

Solution the relationship (3.3) is a vector of unknowns $\hat{\mathbf{X}} = [\hat{\mathbf{s}}^j, \hat{\boldsymbol{\varepsilon}}^j]$ with the following form

$$\hat{\mathbf{X}} = \begin{bmatrix} \hat{\mathbf{s}}^j \\ \hat{\boldsymbol{\varepsilon}}^j \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2 \\ \mathbf{A}_2^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_2^T\mathbf{P}\mathbf{A}_2 \end{bmatrix}^- \begin{bmatrix} \mathbf{A}_1^T\mathbf{Pl} \\ \mathbf{A}_2^T\mathbf{Pl} \end{bmatrix} \tag{3.4}$$

where notation $[\bullet]^-$ denotes g – inverse of the matrix.

Determination of the covariance matrix of adjusted parameters $\mathbf{C}_{\hat{\mathbf{X}}} = \mathbf{D}\mathbf{C}_l\mathbf{D}^T$ ($\mathbf{C}_l = m_0^2\mathbf{P}^{-1}$), for $\mathbf{D}= \begin{bmatrix} \mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2 \\ \mathbf{A}_2^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_2^T\mathbf{P}\mathbf{A}_2 \end{bmatrix}^- \begin{bmatrix} \mathbf{A}_1^T\mathbf{P} \\ \mathbf{A}_2^T\mathbf{P} \end{bmatrix}$ results in

$$\mathbf{C}_{\hat{\mathbf{X}}} = m_0^2 \begin{bmatrix} \mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2 \\ \mathbf{A}_2^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_2^T\mathbf{P}\mathbf{A}_2 \end{bmatrix}^- \tag{3.5}$$

where the matrix of cofactors $\mathbf{Q}_{\hat{\mathbf{X}}}$ has the following form

$$\mathbf{Q}_{\hat{\mathbf{X}}} = \begin{bmatrix} \mathbf{A}_1^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_1^T\mathbf{P}\mathbf{A}_2 \\ \mathbf{A}_2^T\mathbf{P}\mathbf{A}_1 & \mathbf{A}_2^T\mathbf{P}\mathbf{A}_2 \end{bmatrix}^- \tag{3.6}$$

($m_0^2 = \mathbf{v}^T\mathbf{P}\mathbf{v} / f$, $f = n\text{-}r\text{+}d$ – the number of degrees of freedom; n – number of observations, r – number of unknowns, d – network defect).

4. An Example of Practical Application

The following experiment was conducted in order to verify the proposed method of adjustment. From the measurements of a III class net, conducted by GPS technology at Łomianki (Z. Rzepecka, A. Wasilewskiet al., 1995), a fragment (Fig. 2) has been selected and used for thorough practical testing of the proposed method of determination of displacements and strains of terrain.

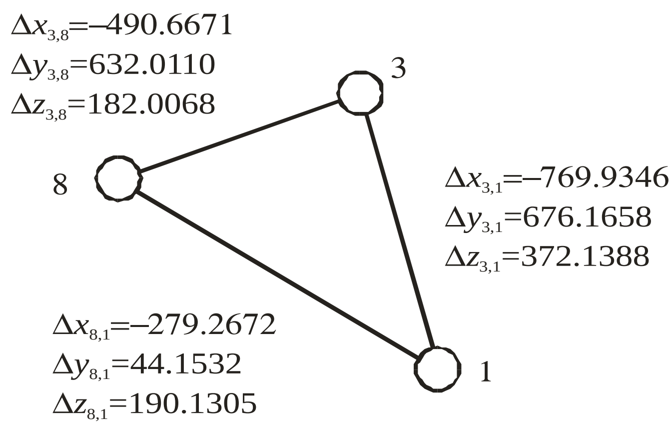

Figure 2

The following vectors are the sought parameters

$\mathbf{s} = \left[S_{X_1} \;\; S_{Y_1} \;\; S_{Z_1} \;\; S_{X_3} \;\; S_{Y_3} \;\; S_{Z_3} \;\; S_{X_8} \;\; S_{Y_8} \;\; S_{Z_8} \right]^T$, and $\quad \boldsymbol{\varepsilon}^T = [\varepsilon_X \;\; \varepsilon_Y \;\; \varepsilon_Z]^T$.

The components of vectors $\Delta x, \Delta y, \Delta z$ shown in Fig. 2, have been treated as the initial measurement and adjusted by the method proposed in this paper. Subsequently, simulating the movements of the surface under study by the values presented in Table 1, the data were transformed with the use of the relationship (2.2). This produced four variants of the network

Variant number	Parameters of transformation					
	T_X [m]	T_Y [m]	T_Z [m]	ε_X^{cc}	ε_Y^{cc}	ε_Z^{cc}
I	0.01	0.02	-0.03	2.54	1.91	1.27
II	0.01	0.02	0.03	0.25	0.19	0.13
III	0.01	0.02	0.03	0.02	0.02	0.01
IV	-	-	-	0.02	0.02	0.01

Table 1

Following the transformation of coordinates, for each of the analysed variants, the vectors components $\Delta x^j, \Delta y^j, \Delta z^j$ were calculated, producing the “current measurement results”.

Tables 2 and 3 show the results of adjustments, obtained in the calculations. Table 2 shows the angles of rotation $\hat{\varepsilon}_{\hat{X}}^{cc}, \hat{\varepsilon}_{\hat{Y}}^{cc}, \hat{\varepsilon}_{\hat{Z}}^{cc}$ and their mean errors, as well as the differences between the results produced in particular variants and the primary result.

Variant number	Estimation results					
	$\hat{\varepsilon}_{\hat{X}}^{cc}$	$\hat{\varepsilon}_{\hat{Y}}^{cc}$	$\hat{\varepsilon}_{\hat{Z}}^{cc}$	$\hat{\varepsilon}_{\hat{X}}^{W} - \hat{\varepsilon}_{\hat{X}}^{P}$	$\hat{\varepsilon}_{\hat{Y}}^{W} - \hat{\varepsilon}_{\hat{Y}}^{P}$	$\hat{\varepsilon}_{\hat{Z}}^{W} - \hat{\varepsilon}_{\hat{Z}}^{P}$
	$m_{\hat{\varepsilon}_{\hat{X}}}^{cc}$	$m_{\hat{\varepsilon}_{\hat{Y}}}^{cc}$	$m_{\hat{\varepsilon}_{\hat{Z}}}^{cc}$			
Initial measurement	-0.05	0.04	0.04	-	-	-
	1.02	1.02	0.57			
I	35.32	-1.61	8.80	35.37	1.57	8.76
	22.87	21.55	12.03			
II	8.29	21.55	18.82	8.34	22.32	18.78
	14.20	9.42	7.58			
III	8.01	22.34	18.73	8.06	22.30	18.69
	14.07	13.43	7.51			
IV	-0.05	0.09	0.05	0.00	0.05	0.01
	1.00	0.96	0.53			

Table 2

P – initial measurement

W –discussed variants

An analysis of the values shown in Table 2 reveals that variants I, II and III, which represent the current measurements, the differences in the angles of rotation assume the values within the interval <1.57^{cc}, 35.37^{cc}>. Therefore, the conclusion can be drawn that the net points did not retain stability in the assumed reference system and were displaced. In variant IV, only the difference $\hat{\varepsilon}_{\hat{Y}}^{W} - \hat{\varepsilon}_{\hat{Y}}^{P}$ allows for suspicions regarding the displacement of the net points.

In Table 3 are showing components of the vector strains (differences between coordinates).

Variants	Strains								
	$\Delta\hat{X}_1^{W-P}$	$\Delta\hat{Y}_1^{W-P}$	$\Delta\hat{Z}_1^{W-P}$	$\Delta\hat{X}_3^{W-P}$	$\Delta\hat{Y}_3^{W-P}$	$\Delta\hat{Z}_3^{W-P}$	$\Delta\hat{X}_8^{W-P}$	$\Delta\hat{Y}_8^{W-P}$	$\Delta\hat{Z}_8^{W-P}$
I	0.0134	-0.0006	-0.0095	-0.0006	0.0015	0.0045	-0.0129	-0.0009	0.0096
II	0.0069	0.0006	-0.0047	-0.0003	0.0013	-0.0002	-0.0060	-0.0019	0.0048
III	0.0062	0.0006	-0.0047	-0.0003	0.0013	-0.0002	-0.0060	-0.0019	0.0048
IV	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3

The results of adjustment shown in Table 3 show that surface strains occurred in variants I, II, III, whereas the results of calculations presented in variant IV do not indicate the possibility of any strains.

The data shown in the tables allow for certain generalisations concerning the analysed example. The results shown in variants I, II and III indicate that a displacement of the net points and strain of the surface under study have occurred. The results obtained in variant IV indicate that only displacement have occurred in the study area.

5. Conclusions

The example presented above does not provide grounds for general conclusions. The results obtained from calculations with the use of the proposed method are an encouragement to conduct further, more detailed theoretical and empirical studies.

Reference

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