ESTIMATION OF THE VERTICAL DEFORMATIONS OF THE STYLOBATE OF ANCIENT TEMPLES - THE CASE OF THESEION

Dr. George GEORGOPOULOS and Dr. Elisavet TELIONI

Rural & Surveying Engineering School, Department of Topography
National Technical University of Athens, Greece

Abstract: In this paper a method is presented for the estimation of the vertical deformations of the stylobate of ancient temples. The surface of the stylobate – the upper surface of the temple's crepis- is a curved one with different transversal and longitudinal gradients. Its original shape can be derived from the uneven heights of the columns' lowermost drums. Since these drums are not affected by distortional features through out the centuries, their heights, if measured accurately, can be used for the determination of the original shape of the temple's stylobate. Using least squares techniques the best fitting surface can be determined. Finally the deformations of the stylobate can be estimated from the comparison between the estimated best fit surface and the nowadays profiles of the stylobate. The settlements of the stylobate of the temple of Hephaestus (Theseion) are presented as a case study. The original curved profiles of the four sides of the monument are approximated by a best fitting line , and the settlements of the temple's stylobate are estimated.

1. INTRODUCTION

One of the main problems in restoration of ancient monuments (temples, theatres etc.) is that their original form (shape and dimensions) is not known. Their parts have deformed throughout the centuries up to nowadays, due to various causes of natural as well as human effect, and, obviously, the "original plans" do not exist.

However, the knowledge of the original form of the ancient monument under consideration is of crucial importance for a correct restoration program, since it is through the comparison of the form the monument has nowadays (actual form) with the estimated original one (ancient form), that the deformations of the monument's parts can be determined. Consequently, the way the restoration will evaluate depends enormously on the correct approximation of the original figure of the monument.

In this paper a method for the estimation of the shape of the stylobate of ancient temples is presented. The method is based on the measurement of the uneven heights of the lowest drums of the columns of the peristasis (the peristyle) of the temple.

2. STRUCTURE AND STYLE OF AN ANCIENT TEMPLE

Ancient Greek architects strove for the precision and excellence of workmanship. It is the Greek temple that best exemplifies the aims and methods of Greek architecture. The most important innovation of the Greek architecture was the external fluted colonnade, *the peristasis*. It formed a "curtain" around the temple, thus screening the sanctuary and the cult statue from the outside world.

The temple consisted of a rectangular room (*the cella*) with projecting walls framing a porch (*pronaos*) at one end. The building stood on top of a stepped platform – the crepidoma - consisting of the euthynteria, the 1st and 2nd steps and *the stylobate*, upon which the columns are erected. The fluted columns consisted of three parts: the base, the shaft and the capital, and they supported an entablature. The entablature consisted of the *architrave* and the *frieze*, composed of alternating *metopes* and *triglyphs*. Above the entablature was a low roof decorated with moulded ends (*cornices*). The triangular space at each end of the roof (*the pediment*) was often decorated with statues.

The ancient Greek temples were usually divided into two principal orders: the Doric and the Ionic one, their difference being mainly on the shape and form of their columns. Thus, the Doric fluted columns had no bases and very simple capitals, they were composed of a number of superimposed drums, held together by wooden or bronze dowels. The ratio between height and diameter was about 6:1. In order to counteract the optical illusion that makes straight sided shafts look slightly concave, the columns were made in such a way that they bulged in the middle. This effect is known as *entasis*.

The capital has developed from simple slabs used as cushions, in order to allow the builders to compensate for differences in level from column to column. It consisted of two elements, usually carved from the same block: the *echinus* (the top part of the upper shaft), which spreads as it rises in order to make a smooth transition to the overlying *abacus*. Since the early builders were uncertain of the load bearing capacity of the stone beams, which made up the architrave, they kept them fairly short. Consequently the columns had to be closely spaced. These strong vertical lines deceived the eye, into making the sides of the temple appear to sag in the middle. To compensate for this, the architects made the architrave and the stylobate, upon which the columns rested, slightly concave.

The Ionian columns have more slender proportions (the ratio of height to diameter is as much as 8:1) and have more extensive and more detailed fluting. The flutes are separated by flat *arrises* rather than sharp ones. The column bases consist of an upper, convex part known as a *torus* and a lower, cylindrical part known as a *spira*. Ionic capitals consisted of a very narrow *echinus* surmounted by a scrollwork *volute* and a relatively small *abacus.*

Thus, slowly by slowly, the vertical structure of the temple conformed to an order, a fixed arrangement of forms, unified by principles of symmetry and harmony. The architectural order governs not only the column, but also the relationships between all the components of the construction.

3. THE REFINEMENTS OF THE GREEK TEMPLES

The most evident proof of the relationships that exist between the components of a Greek temple, is the correlation between the curvature of the stylobate and the inclination of the columns of the peristasis: they are constructed in the most refined way.

The term *refinements* is used to describe the slight adjustments of the horizontal and vertical lines of the Greek temples. These adjustments, according to Vitruvius, were made in order to compensate for optical illusions, while modern scientists draw attention to the fact that refinements give the temple an impression of being a living mass that responds to its own weight.

As it has been mentioned above, the surface of the stylobate, as well as all the other surfaces of the crepidoma, was not horizontal but a curved one, with different transverse and longitudinal gradients. These outwards inclinations not only facilitated the shedding away of the rainwater, but they were also used to counteract distortions of human visual perception: the ancient architects had realized that long horizontal lines tend to make the optical impression of sagging towards the center. To prevent this effect the stylobate and the entablature were constructed curved. The curves of the stylobate are formed in such a way that, the transition from the inclined level of one side to the inclined adjoining one is made with the most refined manner. Curves with nearly regular curvature are used, set in such a way that the tangents at the two ends have the same gradient with the transverse gradient of the stylobate of the adjoining sides.

Because of the convex surface of the stylobate the columns of the peristasis are not vertical, since this would produce the optical illusion that they incline outwards. In order to compensate for this, the columns' axes are delicately inclined inwards with a rate equal to the outwards inclination of the stylobate. Although the exterior generatrices of the columns have an inwards incline the interior ones are perpendicular to the stylobate. Thus, if somebody stands outside the temple, he has the impression of a pyramidal convergence of the columns; while, for the visitor who stands in the pteron, the columns look and are vertical.

In order to blur the pronounced difference between the inclinations of the inner and outer generatrices of the columns, they were in their turn refined by a pronounced "swelling" (*entasis*) of the shaft, and, consequently, the inward inclination of the column is aesthetically rectified. On the other hand the intercolumnium (the gap between the columns) at the corners of the temple and the adjacent columns is slightly smaller than the intercolumnia of all the other columns. The reason is that, otherwise, the corner columns would look quite isolated, moreover, in this way, the corner triglyph is situated right above the axis of the column. The corner columns are slightly thicker than the other ones because they are viewed against the bright sky so, they look thinner than they really are. Their axes, inclining towards the diagonal, have an inclination greater than that of the other columns.

4. DETERMINATION OF THE ORIGINAL CURVATURE OF THE STYLOBATE

The original curvature of the stylobate can be determined by measuring, using accurate methods, the uneven heights of the columns' lowermost drums, which alone have not been affected by distortional features. The fact that the heights of the lowermost drums are uneven, is the most forceful proof that the stylobate was constructed curved on purpose, and that its curvature, identified by levelling in many ancient temples, is not due to deformations.

For the lowermost drum of every column the following relationships exist between the heights of the drum and the inclinations of the stylobate:

The ratio of the difference between the heights h_F^i and h_B^i, measured at the front and back axial flute of i column's drum respectively, to the diameter D_C of the drum's base, equals the transverse inclination $k^i_{Axis,Transverse}$ of the column's axis with respect to the stylobate. This inclination is the sum of the inclination of the column's axis k^i_{Axis} with respect to the plumb line plus the transverse inclination $k^i_{S,Transverse}$ of the stylobate:

$$k^i_{Axis,\ Transverse} = \frac{h^i_F - h^i_B}{D_C} = k^i_{Axis} + k^i_{S,\ Transverse} \qquad (1)$$

When the columns are observed sideways they incline inwards. On the contrary they are vertical when observed from the front. Therefore the horizontality of the front seam of the upper side of the drum is achieved through the uneven heights at the left and right axial flute – h^i_L and h^i_R respectively. The ratio of the difference between these heights to the diameter D_C of the drum's base equals the longitudinal inclination $k^i_{S,Longitudinal}$ of the stylobate, at the position of column i :

$$k^i_{S,Longitudinal} = \frac{h^i_L - h^i_R}{D_C} \qquad (2)$$

Although the transverse inclinations of the stylobate are nearly constant, at least for each side, the longitudinal one changes having its maximum values at the corners and the minimum ones at the middle of each side.

The profiles of the four sides of the original stylobate can be plotted in development, with respect to one of the corners of the stylobate. The ordinates of the profiles are determined using the calculated longitudinal inclinations at the position of each column, while the abscissas are the distances of each column from the reference corner. These profiles are, of course, the cross –sections of the stylobate's surface with a vertical plane at the position of the columns' axes. Therefore they are polygonal lines with sides the couples of stones on which the columns stand, (except for the corner columns that stand on a single stone), as well as the stones of the intercolumnia. However, the breakings of these lines are so imperceptible that it can be assumed to be curves.

Using least squares techniques a best fitting curve –describing the original curve can be estimated, using as observations in the adjustments the ordinates of the stylobate's profiles.

The nowadays curve of the stylobate can be estimated through precise leveling at especially selected positions. The profiles of the four sides of the actual stylobate can be compared to the original ones, and the (possible) deformations of the stylobate estimated.

5. A CASE STUDY – THE STYLOBATE OF THE THESEION TEMPLE IN ATHENS

The method described above has been applied for the estimation of the vertical displacements of the stylobate of the Theseion temple in Athens.

5.1. The Temple

The temple of Hephaestus and Athena Ergane, also known as Theseion is a Doric order temple located at the northwest side of the ancient Agora of Athens, in a district which contained many foundries and metalwork shops. Its construction started at 449 BC, but was not completed before 415 BC, possibly because funds and workers were directed towards the construction of the monuments of the Athenian Acropolis. It's architect is not known, but is assumed to be the same who designed the temple of Poseidon at cape Sounion.

The temple is hexastyle i.e. with six columns under the pedimented ends (short, east and west, sides) and thirteen columns at the long, north and south, sides. It is a peripteral temple, with columns entirely surrounding the central enclosed cella. Pentelic marble was used for the construction, with the exception of the lowest step of the crepidoma which is from limestone. The building has a pronaos, a cella and an opisthodomos.

Unlike the Parthenon, the temple has all its columns and pediments intact, and even has most of its original roof. However its friezes and other decorations have inevitably been damaged over the centuries. It owes its survival to its conversion to a Christian church in the 7th century AD.

5.2. Determination of the original curves of the stylobate's four sides

The uneven heights of the lowermost drums of the columns of the peristasis were measured using a pachymeter having an accuracy of ±0.1mm. The heights were measured at the front, back, left and right axial flute of each column's lowest drum.

From the right and left axial heights the longitudinal inclinations of the stylobate stones, that bear the columns, are estimated, using Formula 2. The diameter of the drum's base being $D_C = 0.954$m. The longitudinal inclinations take their maximum values at the corner stones (≈0.6%, except for the SW corner where the inclination is 0.46%), and their minimum ones at the midpoint of each side.

From the calculated longitudinal inclinations the ordinates of the centers of the drums' bases were determined (Table 1). Using as abscissas the distances of the centers from the NE corner, the profiles of the four sides of the original, undistorted stylobate were plotted in development and are depicted in Figure 2. As it can be seen the four corners of the temple are not located at the same level. The lowest one is the SW corner, and with respect to that, the NE one is +5mm, the NW +20mm, and the SW +10mm higher. It must be pointed out that The same fact is observed in many other ancient temples (Parthenon, temple of Zeus in Nemea etc). The rises of the curves of the four sides of the original curves are : 20mm on the east side, 25mm on the north, 20mm on the west and 30mm on the south.

For each side of the stylobate the parabolic curve that fits best its actual profile was estimated using as observations the ordinates of the centres of columns that belong to this side.. For the centre of column *i* the following observation equation was formed:

$$a \cdot x_i^2 + b \cdot x_i + c = y_i + \upsilon_i \qquad (3)$$

where: a, b and c: the coefficients of the curve, treated as the unknown parameters,
x_i : the centres' distances from the left corner of the side under consideration,
y_i: the centres' ordinates treated as observations in the adjustment, and
υ_i: the residuals in the adjustment.

After the adjustment, the estimates of the unknown parameters $\hat{a}$, $\hat{b}$ and $\hat{c}$ were determined together with their a posteriori variance covariance matrix $\hat{V}_{\hat{x}}$.

The following equations giving the best fit curves were obtained from the four independent least squares adjustments, one for each side of the stylobate:

Northern side

$$y = -1.10 \cdot 10^{-4} x^2 + 3.66 \cdot 10^{-3} x + 2.78 \cdot 10^{-2} \tag{4}$$

Western side

$$y = -4.44 \cdot 10^{-4} x^2 + 6.89 \cdot 10^{-3} x + 3.03 \cdot 10^{-2} \tag{5}$$

Southern side

$$y = -9.77 \cdot 10^{-5} x^2 + 2.39 \cdot 10^{-3} x + 4.44 \cdot 10^{-2} \tag{6}$$

Eastern side

$$y = -4.05 \cdot 10^{-4} x^2 + 5.92 \cdot 10^{-3} x + 2.18 \cdot 10^{-2} \tag{7}$$

The ordinates $\hat{y}_i$ of the columns' centers were calculated using the estimated best fit curves, and were compared to those determined through the measured longitudinal inclinations (Table 1), in order to examine the goodness of fit. As it can be seen for most of the centers the departures do not exceed ±1mm, while the maximum ones are observed in the southern side of the stylobate, but do not exceed 3mm. Therefore, the parabolic curves estimated above can be used to represent the original curves of the four sides of the stylobate.

These smoothed curves are plotted against the measured ones and depicted in Figure 2.

5.3. Estimation of the vertical displacements of the stylobate

The actual curves of the stylobate were determined by precise geometric leveling. For every column, the heights of the four points of the stylobate corresponding to the four axial flutes of the column were measured, and the heights of the columns' centers were estimated with an accuracy of ±2mm. The results are depicted in Table 1, and the plotted profiles of the actual curves of the stylobate appear in Figure 2.

Following, the vertical displacements of the stylobate at the position of the columns' centers were estimated and their statistical significance was tested. For a significance level of 95% all vertical displacements having magnitude greater than 7mm were considered as statistically significant.

Statistically significant subsidences ranging from 8mm up to 41mm are observed. The most important ones are detected in the northern side (22mm - 41mm), but the other three sides of the stylobate have also deformed.

These observed subsidences are due to insufficient foundation of the temple, as well as to deformations caused by earthquakes, and human effects. Digging graves around the temple and in the cella, during the centuries after the prevalence of Christian religion, has harmed the foundation of the temple. The foundation remained uncovered for more than three centuries, before the beginning of the restoration works by the American School of Classical Studies in 1936. Therefore although the stylobate of the temple was constructed convex, its actual shape is due to extended settlements.

6. CONCLUSIONS

From the method described above and the case study it is shown that the geometric features of the stylobate original curves can be estimated through the measurement of the uneven heights of the lowermost drums of the columns. These curves can be approximated by analytical methods such as parabolic equations, using least squares techniques for the coefficients estimation. The differences between the geometric features determined analytically and those measured in the field are within the overall accuracy of the measurement method.

The (possible) displacements of the stylobate of ancient temples can therefore be estimated by comparing the actual convex surface of the stylobate, and the one determined analytically. It is however important to use measuring methods of equal precision.

The method should be applied in other Greek temples of Doric as well as Ionian order for the investigation of the temples’ characteristics and possible differences between the orders.

SIDE	COLUMN	x_i (m)	y_i (m)	$\hat{y}_i$ (m)	$y_{levelling}$ (m)	Δy_i (m)
	NE corner	0.000	0.026	0.026		
	NEC	0.558	0.029	0.030	0.000	**-0.030**
	NC2	2.986	0.040	0.038	0.010	**-0.028**
	NC3	5.561	0.046	0.045	0.012	**-0.033**
N	NC4	8.147	0.051	0.050	0.023	**-0.027**
O	NC5	10.730	0.055	0.054	0.031	**-0.024**
R	NC6	13.310	0.056	0.057	0.035	**-0.022**
T	NC7	15.889	0.056	0.058	0.037	**-0.022**
H	NC8	18.443	0.056	0.058	0.034	**-0.024**
	NC9	21.050	0.056	0.056	0.031	**-0.025**
	NC10	23.632	0.054	0.053	0.029	**-0.024**
	NC11	26.211	0.050	0.048	0.023	**-0.025**
	NC12	28.793	0.045	0.042	0.011	**-0.031**
	NWC	31.198	0.034	0.035	-0.006	**-0.041**
	NW corner	31.771	0.030	0.032		
	NWC	32.336	0.034	0.034	0.002	**-0.032**
W	WC2	34.749	0.047	0.047	0.017	**-0.030**
E	WC3	37.332	0.053	0.055	0.028	**-0.027**
S	WC4	39.915	0.058	0.057	0.031	**-0.026**
T	WC5	42.497	0.054	0.053	0.030	**-0.023**
	SWC	44.937	0.044	0.044	0.026	**-0.018**
	SW corner	45.487	0.041	0.043		
	SWC	46.037	0.044	0.046	0.026	**-0.020**
	SC12	48.460	0.052	0.051	0.036	**-0.015**
	SC11	51.047	0.058	0.055	0.040	**-0.015**
	SC10	53.623	0.061	0.057	0.045	**-0.012**
S	SC9	56.200	0.061	0.059	0.049	**-0.010**
O	SC8	58.784	0.059	0.059	0.051	**-0.008**
U	SC7	61.364	0.056	0.058	0.053	-0.005
T	SC6	63.948	0.053	0.055	0.051	-0.005
H	SC5	66.530	0.049	0.051	0.049	-0.003
	SC4	69.106	0.045	0.046	0.043	-0.004
	SC3	71.749	0.040	0.040	0.033	-0.007
	SC2	74.262	0.034	0.032	0.025	**-0.008**
	SEC	76.718	0.025	0.024	0.015	**-0.009**
	SE corner	77.248	0.022	0.022		
	SEC	77.863	0.026	0.025	0.014	**-0.012**
E	EC5	80.235	0.036	0.036	0.022	**-0.014**
A	EC4	82.817	0.041	0.042	0.028	**-0.015**
S	EC3	85.402	0.043	0.043	0.028	**-0.016**
T	EC2	87.989	0.040	0.039	0.024	**-0.015**
	NEC	90.379	0.030	0.030	0.009	**-0.021**
	NE corner	90.974	0.026	0.027		

Table 1 - Estimated vertical displacements of the Theseion stylobate. The significant settlements are depicted with bold characters.

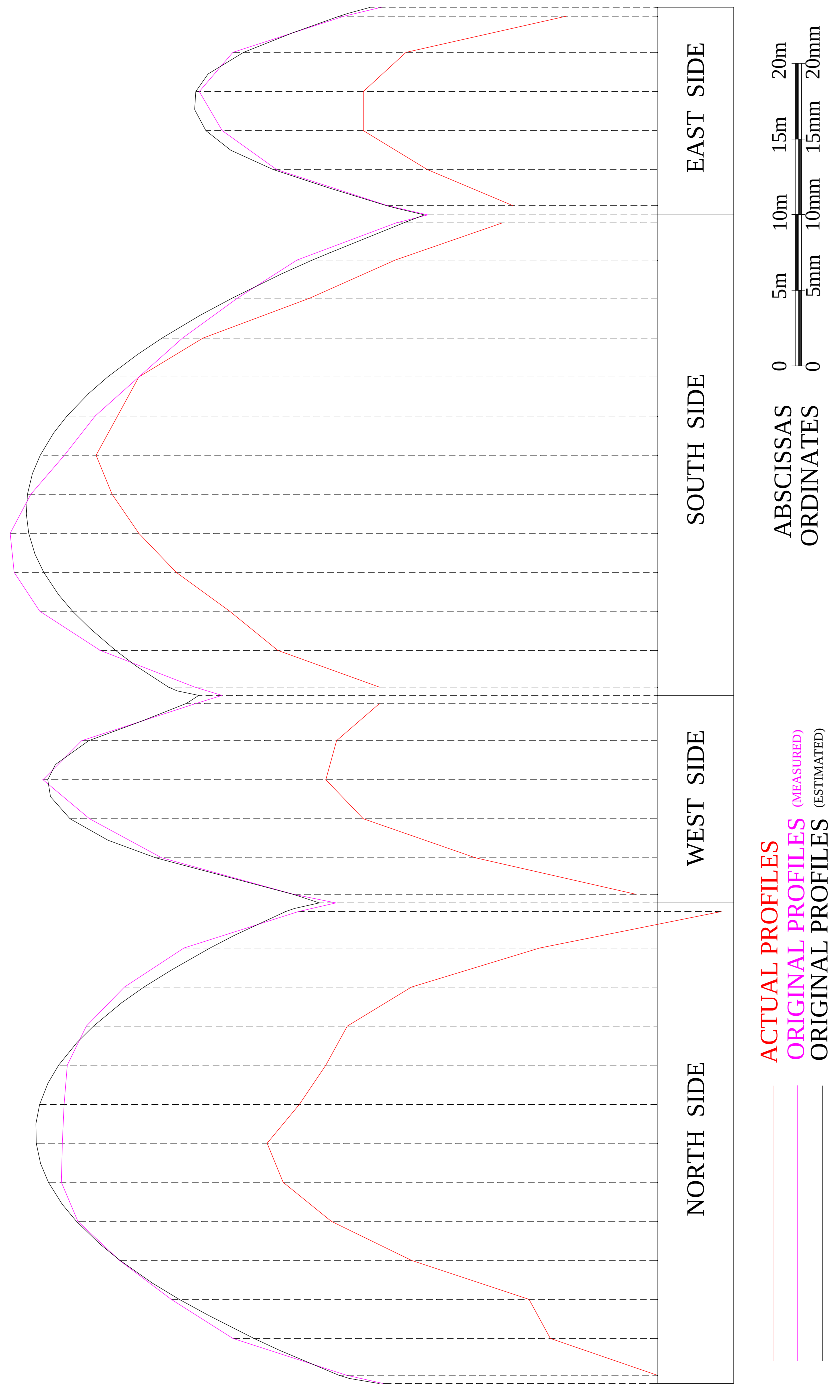

Figure 1- Profiles of the four sides of the stylobate of Theseion.

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Corresponding author contacts

George D. Georgopoulos
gegeorgo@central.ntua.gr
National Technical University of Athens
Greece